Chapter 16.

MODELS FOR ASSESSING THE FINANCIAL STABILITY OF INSURANCE COMPANIES

The efficient functioning of the insurance market requires financially reliable insurers, able to fulfil their obligations to policyholders on time and in full. The financial stability of insurers is the starting point for the continued implementation of the insurance function. The frequency and variety of risk manifestation models necessitate thorough risk analysis to justify the adoption of financial strategies by insurance companies. The application of economic-mathematical models provides a reliable basis for assessing the financial stability of insurers. To achieve financial stability, insurance companies engage in mathematical modelling of key indicators such as the probability of bankruptcy and the solvency margin.

Modelling the probability of bankruptcy in insurance companies serves as a tool for preventing insolvency. Accordingly, determining an adequate insurance premium is based on a previously estimated probability of bankruptcy. Differences between models arise from assumptions regarding the distribution, amount, and timing of insurance claim payments, which form the basis for their construction. Insurance claim payments are modelled using an appropriate probability distribution. The time intervals between claim payments are typically modelled using exponential distributions, while the sequence of claim events is defined by a Poisson process.

Assessing the financial stability of an insurance company is a complex process due to the multifaceted nature of the issue. The use of economic-mathematical models provides a foundation for generating valuable insights in the management decision-making process. From a market perspective, insurance companies aim to achieve the most reliable assessments of financial stability. However, this assessment is highly complex, and existing mathematical models often fail to incorporate all external factors that affect the financial soundness of insurance companies. Including a large number of variables results in a highly complex model that requires sophisticated solutions but yields more precise results. Conversely, simplifying the model by excluding certain factors facilitates easier computation but reduces the accuracy of the outcome.

Many factors affecting financial stability are difficult to express through explicit analytical relationships, which necessitates the use of approximate or simulationbased models. The aforementioned models serve to assess the financial stability of companies, though they remain subject to the previously mentioned limitations in their development. Their application significantly enhances the validity of management decision-making and supports the financial stability of the company, based on indicators such as the probability of bankruptcy, the amount of initial capital and technical reserves, the solvency margin, and others. The initial capital and technical reserves of insurance companies operating in the Republic of Serbia are regulated by the Law on Insurance and the regulations of the National Bank of Serbia (NBS). The financial performance of insurance companies is published in NBS reports, which serve as one of the criteria for ranking and selecting companies within the insurance market.

Initial capital represents the minimum amount of a company's core capital, the required levels of which are prescribed by the Insurance Law depending on the type of insurance activity performed.[469] Insurance companies are obliged to form technical reserves to ensure the fulfilment of contractual obligations toward policyholders. The company's technical reserves include:

- *Outstanding claims reserves,* which refer to claims that have occurred and for which the insurer is liable, but that have not been settled as of the balance sheet date.
- *Premium reserves,* representing the portion of the premium allocated to cover future risks during the insurance period.
- *Mathematical reserves,* applicable to life insurance, which represent the present value of future obligations arising from concluded contracts.
- *Reserves for claims oscillation,* intended to cover unexpected fluctuations in claims.
- *Other technical reserves,* formed based on the current balance sheet and prescribed in accordance with the regulations of the National Bank of Serbia (NBS).

Technical reserves must be adequately backed by the company's assets to ensure both solvency and liquidity. The National Bank of Serbia oversees the formation of technical reserves in order to protect policyholders and maintain stability in the insurance market.

The financial stability of an insurance company is analysed using two models: the first is based on risk analysis through a static approach, while the second uses a dynamic approach. In the static approach, time dependencies are not

[469] Law on Insurance, *Official Gazette of the Republic of Serbia,* No. 139/2014 and 44/2021.

considered, which affects the modelling of premium and claim payments. In contrast, the dynamic approach incorporates time as a variable, meaning that the payment of insurance premiums and settlement of claims is time-dependent.

Reserve adequacy refers to an assessment of whether the allocated funds are sufficient to ensure the company's solvency. This evaluation is performed using a probability model or a marginal probability model, which measures the likelihood of a particular variable occurring independently of others. Thus, reserve adequacy is assessed using probabilistic models that estimate the likelihood that collected premiums will be sufficient to cover total claims in the observed period. An advantage of this approach lies in the simplified estimation of distribution parameters for random variables associated with individual insurance risks. To assess overall portfolio risk, claim payments are compared to received premiums and established reserves.

1. EVALUATION OF ECONOMIC-MATHEMATICAL MODELS FOR INSURANCE RISK ANALYSIS

Risk model based on a static approach

The static approach to risk modelling assumes that the total amount of claim payments is the sum of individual insurance claims paid to separate policyholders, forming the insurer's portfolio. The total claim amount is modelled as the sum of random variables representing actual claim payments. A key criterion in selecting an insurance company is the ratio between the insurance premium and the insured sum[470].

The static model considers the insurer's portfolio under the following assumptions:

– A set of insured risks is established at a single point in time.
– Insurance premiums are paid at the moment the portfolio is formed.
– The duration of all insurance contracts is identical.
– The conclusion of an insurance contract entitles the policyholder to potential indemnity.

[470] Popović, Z., Backović, M., & Babić, S. (2016). Risk analysis in life insurace policy selection by applying optimization criteria. In: *Risk Management in the financial services sector*, Kočović, J. et al. (eds.), Belgrade: University of Belgrade, Faculty of Economics and Business, pp. 399-417.

The static model is used to calculate the probability of company bankruptcy under the following assumptions:

- A fixed and relatively short time horizon is considered.
- The number of the insurance contracts is known and fixed (NIC).
- Insurance premiums are paid upon contract conclusion (IP).
- Statistical characteristics of individual claim amounts are known (AIC).

The mathematical model describing the financial condition of the insurance company is:

$$IF = IIC + TAIP - TAIC \,. \quad (1)$$

Here, the symbol IF represents the insurance fund at the end of the contract period, while IIC denotes the initial capital of the insurance company. The insurance premiumvis assumed to be identical across all contracts and the total insurance premium denoted as $TAIP$ is calculated using the formula:

$$TAIP = IP * NIC \,.$$

If the premium amount varies across contracts, the following expression is applied:

$$TAIP = \sum_{i=1}^{NIC} IP_i \,,$$

where IP is the premium paid by policyholders to the insurance company. The symbol $TAIC$ denotes the total claim amount under insurance contracts, calculated as:

$$TAIC = \sum_{i=1}^{NIC} AIC_i \,. \quad (2)$$

The following assumptions are made to develop the static model:

- The random variables $AIC_1, \ldots, AIC_{NIC}$ are independent, meaning that risks are not simultaneously covered under multiple contracts.
- These variables are non-negative and predetermined.
- Policyholders are homogeneous, and the random variables $AIC_1, \ldots, AIC_{NIC}$ are identically distributed.
- Not all contracts generate claims, so some of the random variables $AIC_1, \ldots, AIC_{NIC}$ may equal zero.

For the mathematical model of the company's financial position in expression (1), the probability of bankruptcy is given by:

$$PB = P\left\{\sum_{i=1}^{NIC} AIC_i - IIC - IP \cdot NIC \geq 0\right\}. \quad (3)$$

This represents the probability that total claims will exceed the sum of initial capital and collected premiums.

Risk model based on a dynamic approach

A non-decreasing series of random variables $t_0 = 0 \leq t_1 \leq ..., T$ is analysed through the development of a risk model based on a dynamic approach, which characterises the timing of premium payments and the occurrence of claims.

The mathematical model of the financial balance of an insurance company based on a dynamic approach is represented by the following expression:

$$IF(t) = IIC + TAIP(t) - TAIC(t), \quad (4)$$

where:

$TAIP(t) = \sum_{i=1}^{NIC(t)} IP_i$ - represents the total insurance premium at the beginning of period (t), while the symbol $NIC(t)$ refers to the random process representing the number of insurance contracts in period (t);

$TAIC(t) = \sum_{i=1}^{NIC(t)} AIC_i(t)$ - represents the random process of the total amount of claims in a period (t), while $AIC_i(t)$ is the number of claims related to the i-th individual insurance event occurring in period (t).

In practice, insurance premiums are generally received more frequently than claims are paid out, and premium amounts are typically smaller than the amounts paid in claims. Insurance premiums follow a continuous deterministic process, characterised by a single parameter — the cash flow rate of premium payments (cr). Accordingly, the total premium amount in period (t) is calculated using the following expression:

$$TAIP(t) = cr \cdot t. \quad (5)$$

The financial balance of the insurance company based on expression (4) is restated as:

$$IF(t) = IIC + cr \cdot t - \sum_{i=1}^{NIC(t)} AIC_i(t). \tag{6}$$

The associated random process, referred to as the "risk process" in economic-mathematical research,[471] is defined as:

$$RP(t) = cr \cdot t - \sum_{i=1}^{NIC(t)} AIC_i(t). \tag{7}$$

The probability of bankruptcy within a given period *t* is more consistent with real-world conditions. For the model expressed in (6), the bankruptcy probability is defined as:

$$PB = P\left[\exists t_0 : 0 < t_0 < T, t_0 = \min\{t : IIC + RP(t) < 0\}\right]. \tag{8}$$

which represents the probability that the available funds of the insurance company during period *t* will not be sufficient to cover claims. The timing of claims arrivals is described through a non-decreasing series of random variables ($t_0 = 0 \le t_1 \le t_2 ..., t_n$). The time interval between two successive claim events is defined by the following expression:

$$T_n = t_n - t_{n-p} \ge 0. \tag{9}$$

It is assumed that claim payments cannot occur simultaneously under multiple contracts, given the number of claims (nc) for the random process of the insurance contract $NIC(t)$ in the period (t). The distribution of claim amounts is defined as:

$$PD_{nc}(t) = P\{NIC(t) = nc\} = P\left\{\sum_{i=1}^{nc} T_i \le t \le \sum_{i=1}^{nc+1} T_i\right\}. \tag{10}$$

A renewal process is established by assuming that the random variables are independent and identically distributed, with a defined distribution function.

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471 Swishchuk, A., Zagst, R., & Zeller, G. (2021). Hawkes processes in insurance: Risk models, applications to empirical data, and optimal investment. *Insurance: Mathematics and Economics*, *101*, pp. 107-124

For further model development, the following hypotheses are introduced:

- Insurance claim payments in two time intervals are independent.
- The random process of the number of insurance contracts $NIC(t)$ in the period (t) is defined as a process with stationary increments, and $NIC(t+1)-NIC(t)$ and $NIC(t+h)-NIC(t)$ follow the same distribution.
- In each short interval, only one claim request occurs (due to the assumption of stationary increments).
- There is an average number of claims $\lambda > 0$, representing the expected number of claims in each interval, based on stationary increments.

Assumption 1) implies that claim payments occur independently. This is a realistic assumption in practice, as the probability of simultaneous claim events is low. A typical example of a renewal process is the Poisson process. The associated random variable follows an exponential distribution with parameter $\lambda > 0$. Therefore, the distribution $NIC(t)$ has the following expression:

$$p_{nc}(t) = e^{-\lambda t} \cdot \frac{(\lambda \cdot t)^{nc}}{nc!}, \quad nc = 0,1,2,... \tag{11}$$

For the Poisson process, the following holds:

1) Expected value (mean): $E[NIC(t)] = \lambda \cdot t, \quad \mu[NIC(t)] = \lambda \cdot t$,
2) Variance: $\sigma^2 = \lambda \cdot t$.

The probability distributions for the expected number of claims (nc) within a single time period is obtained by applying the Excel function POISSON.DIST.

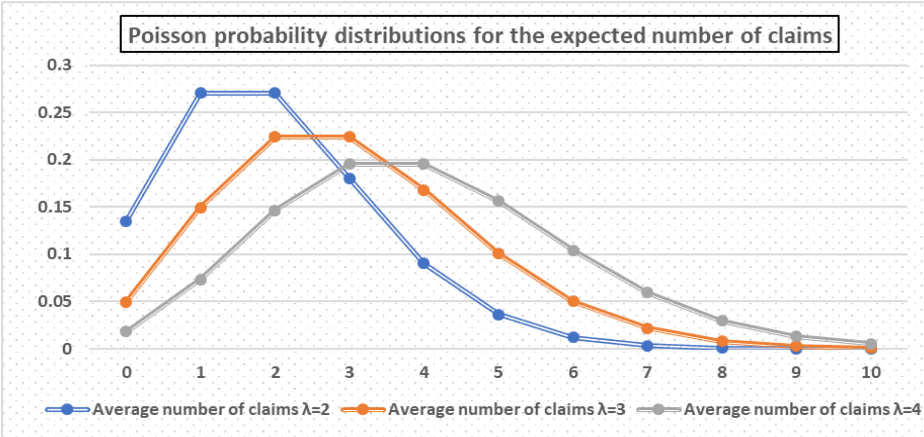
Table 1. Poisson probability distributions

Expected number of claims	0	1	2	3	4	5	6	7	8	9	10
Average number of claims λ=2	0.13	0.27	0.27	0.18	0.09	0.03	0.01	0.003	0.0009	0.0002	4E-05
Average number of claims λ=3	0.04	0.14	0.22	0.22	0.16	0.10	0.05	0.02	0.008	0.002	8E-04
Average number of claims λ=4	0.01	0.07	0.14	0.19	0.19	0.15	0.10	0.05	0.02	0.01	0.005

Source: Author's calculation

Claim amounts are considered random variables and play a central role in risk assessment. A delay between submitting a claim and its settlement is acknowledged; however, the model assumes that claims are paid at the time they are submitted.

Figure 1. Poisson probability distributions



Source: Author's illustration

The claim payment distribution function is defined as:

$$FDPD_{TAIC(t)}(taic) = P\{TAIC(t) \le taic\} = P\left\{\sum_{i=1}^{NIC(t)} AIC_i \le taic\right\}. \tag{12}$$

The assumptions that $\{AIC_i\}$ and $NIC(t)$ are independent random processes are introduced to apply the damage payment distribution function $FDPD_{TAIC(t)}(taic)$. While these assumptions are theoretical, they may not always hold in practice. Independence between processes $\{AIC_i\}$ and $NIC(t)$ is derived from the application of Poisson process. The distribution function for claims by individual insured events $FDPD_{TAIC(t)}(taic)$ (from expression (12)) is represented by the following expression:

$$FDPD_{TAIC(t)}(taic) = P\{TAIC(t) \le taic\} = \sum_{nc=1}^{\infty} p_{nc}(t) \cdot FDPD_{AIC}^{nc}(taic). \tag{13}$$

Based on expressions (7) and (8), the claim distribution function is applied to evaluate the company's financial stability and is expressed as:

$$FDPD_{AIC}^{nc}(taic) = P\{AIC_1 + AIC_2 + \ldots + AIC_{nc} \le taic\}. \tag{14}$$

From an economic perspective, financial stability in insurance companies assumes that claim payments should not deplete available capital. It is essential that the remaining balance of the initial reserve, together with collected premiums, is sufficient to cover all future claims.

2. MATHEMATICAL METHODS AND PRINCIPLES OF CALCULATING INSURANCE PREMIUMS

The insurance premium represents the amount paid by the policyholder to the insurer, either as a lump sum or in instalments, in exchange for coverage against unforeseen events. The process of determining insurance premiums involves assessing the probability and potential impact of insured risks and defining an appropriate compensation function to cover possible losses.

Insurance premiums collected during a given period (t) should, in principle, be sufficient to cover the total claims incurred during that period $TAIC(t)$ - ideally exceeding or at least equalling the expected value of claims. However, theoretical and practical considerations suggest that the determination of insurance premiums is based on several guiding principles. These principles are generally classified into three methodological approaches: the ad-hoc method, the characterisation method, and the economic method.

Some common properties of insurance premium principles include the following:

- *Independence:* A premium principle satisfies the independence property if the distribution function of claim payments $FDPD[TAIC(t)]$ depends solely on the random process of claim amounts $TAIC(t)$ in period (t).
- *Burden of risk:* If the inequality $FDPD[TAIC(t)] \geq E[TAIC(t)]$ holds, the premium principle imposes an unjustified burden of risk. Converserly, if equality applies for all constant risks, the principle is considered not to impose such a burden.
- *Invariance:* This includes both scale invariance and translational invariance. Scale invariance is satisfied if:
  $FDPD[con \cdot TAIC(t)] = con \cdot FDPD[TAIC(t)]$,
  and translational invariance holds if:
  $FDPD[con + TAIC(t)] = con + FDPD[TAIC(t)]$.
- *Additivity:* A principle is additive if it satisfies:
  $FDPD[TAIC(t_1) + TAIC(t_2)] = FDPD[TAIC(t_1)] + FDPD[TAIC(t_2)]$.
  It is sub-additive if:

$$FDPD\left[TAIC\left(t_1\right)+TAIC\left(t_2\right)\right]\leq FDPD\left[TAIC\left(t_1\right)\right]+FDPD\left[TAIC\left(t_2\right)\right].$$

It is super-additive if:

$$FDPD\left[TAIC\left(t_1\right)+TAIC\left(t_2\right)\right]\leq FDPD\left[TAIC\left(t_1\right)\right]+FDPD\left[TAIC\left(t_2\right)\right].$$

One of the key parameters that determines the financial stability of an insurance company is its tariff structure. The tariff rate represents the price of insurance coverage for uncertain future liabilities[472]. This concept is discussed in detail by Kočović et al. (2021). The insurance premium serves multiple functions for insurance companies. It guarantees the payment of future claims and also acts as a benchmark for competition in the insurance market. The ratio between the insurance premium and the insured amount is often used as a criterion for selecting an insurer[473]. The premium calculation method is based on the company's available statistical data. The insurance premium $TAIP(t)$ in the time interval $[t]$ is calculated using the following expression:

$$TAIP(t)=(1+fl)\cdot\left[E\left[NIC(t)\right]-E\left[TAIC(t)\right]\right] \quad (15)$$

where the term (fl) denotes a constant called the *load factor*. It is assumed that the variable $TAIC(t)$ follows the same distribution as each $AIC_i(t)$, which represents the random process of the number of claims related to each insurance case in period (t).

The structure of the insurance premium is based on the principle of equivalence in the relationship between the insurer and the policyholder, as well as on the need to maintain the financial stability of the insurance company. Formula (15) implies that the average insurance premium amounts should exceed the total cumulative value of claim payments during period (t).

Adequate insurance premiums are calculated by constructing a process denoted as $TAIP(t)$, based on the distribution function of claim payments. Accordingly, the key characteristics of the random process representing the total amount of claims $TAIC(t)$ — namely, the mathematical expectation and variance — are used in determining the insurance premium for the claims incurred during period (t). The estimated risk has a significant impact on the insurance premium, as its amount depends on the probability and severity of potential losses, represented

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472 Kočović, J., Rakonjac-Antić, T., Koprivica, M., & Šulejić, P. (2021). *Osiguranje u teoriji i praksi*, Belgrade: University of Belgrade, Faculty of Economics and Business.

473 Popović et al. (2016), op. cit.

by the risk measures $TAIP\left[TAIC\left(t\right)\right]$ or $TAIP\left[FDPD_{TAIC\left(t\right)}\right]$. The symbol $FDPD_{TAIC\left(t\right)}$ denotes the distribution function of claim amounts.

The general properties of the insurance premium function $TAIP\left[TAIC\left(t\right)\right]$ are defined by the following expressions:

- $TAIP\left(con\right) = con$ - for any constant con, when no load factor is applied,
- $TAIP\left[con \cdot TAIC\left(t\right)\right] = con \cdot TAIP\left[TAIC\left(t\right)\right]$ - for any constant con,
- $TAIP\left[TAIC\left(t\right) + con\right] = TAIP\left[TAIC\left(t\right)\right] + con$ - for any constant con,
- $TAIP\left[TAIC_1\left(t\right) + TAIC_2\left(t\right)\right] < TAIP\left[TAIC_1\left(t\right)\right] + TAIP\left[TAIC_2\left(t\right)\right]$,
- $TAIC_1\left(t\right) < TAIC_2\left(t\right) \Rightarrow TAIP\left[TAIC_1\left(t\right)\right] < TAIP\left[TAIC_2\left(t\right)\right]$.

The basic actuarial principles for forming the insurance premium are expressed through the following expressions:

- Expected value principle

$$TAIP\left[TAIC\left(t\right)\right] = \left(1 + con\right) \cdot E\left[TAIC\left(t\right)\right], con > 0 .$$

- Variance (dispersion) principle

$$TAIP\left[TAIC\left(t\right)\right] = E\left[TAIC\left(t\right)\right] + con \cdot Var\left[TAIC\left(t\right)\right] .$$

- Standard deviation principle

$$TAIP\left[TAIC\left(t\right)\right] = E\left[TAIC\left(t\right)\right] + con \cdot \sqrt{Var\left[TAIC\left(t\right)\right]} .$$

- Exponential utility principle

$$TAIP\left[TAIC\left(t\right)\right] = \frac{1}{con} \cdot \log E\left[e^{con \cdot TAIC\left(t\right)}\right].$$

- Absolute deviation principle

$$TAIP\left[TAIC\left(t\right)\right] = E\left[TAIC\left(t\right)\right] + con \cdot k_{TAIC\left(t\right)} .$$

- Equivalent utility (zero utility) principle

$$u\left(0\right) = E\left[u\left(TAIP\left(TAIC\left(t\right)\right) - TAIC\left(t\right)\right)\right] .$$

The risk model analysis is applied to a company whose portfolio consists of insurance contracts $NIC(t)$, with incurred claim payments denoted by $AIC_i(t)$, which are independent and represent a random process for the total amount of claims related to individual insurance events over a specified time period (t). The random process of total claims during period (t) is expressed as: $TAIC(t) = \sum_{i=1}^{NIC(t)} AIC_i(t)$, where $TAIC(t)$ follows a claim distribution function given by: $FDPD_{AIC}^{nc}(taic) = P\{AIC_1 + AIC_2 + ... + AIC_{nc} \leq taic\}$.

The initial assumption implies that the insurance company concludes insurance contracts for a limited period. Each contract covers only one claim event, meaning that multiple claims are not settled under a single insurance contract. The payment for an individual claim is represented by a random variable $AIC_i(t)$ whose value may be equal to zero. Based on these assumptions, the total claim payment at the end of the insurance contract period is given by the following expression: $TAIC(t) = \sum_{i=1}^{NIC(t)} AIC_i(t)$.

The probability of bankruptcy of an insurance company is calculated as $PB\{TAIC(t) > IIC + TAIP(t)\}$, i.e., the probability that the total amount of claim payments exceeds the sum of the initial capital and collected insurance premiums.

The Poisson process is traditionally based on the assumption that all insurance claim amounts are identical. However, in practice, claim amounts vary significantly. Consequently, instead of assuming fixed claim amounts, we incorporate randomly distributed claim values, resulting in a more complex version of the Poisson process.[474] If the random variable representing claim size is not exponentially distributed with parameter $\lambda > 0$, the analytical tractability of the model is lost, and closed-form results become difficult to obtain, demonstrating a key limitation of the standard Poisson process. To estimate the number of claims and assess potential insolvency, approximate results can be derived using the Central Limit Theorem. These approximations serve as useful tools in calculating the probability of an insurance company's bankruptcy.

The probability of bankruptcy is calculated with considerable technical complexity, often requiring approximation methods based on the Central Limit Theorem. The insurance premium is defined as the price of insurance (the price

[474] Challa, A. (2012). *Insurance models and risk-function premium principle.* University of Warwick, pp. 1-23.

of risk), and is used to determine the amount payable by the policyholder.[475] The total insurance premium consists of the net premium and the overhead allowance. The net premium itself is composed of two components: the technical premium and the preventive allowance. The technical premium must be sufficient to cover claims arising under the insurance policy, while the preventive allowance is allocated for activities aimed at reducing and preventing insured losses.

The principle of net insurance premium is defined by the following expression:

$$TAIP(t) = E[TAIC(t)]. \tag{16}$$

The probability of company bankruptcy is given by the equation:

$$PB\{TAIC(t) > TAIP(t)\} = PB\{TAIC(t) - E[TAIC(t)] > 0\} =$$

$$= PB\left\{\frac{TAIC(t)-E[TAIC(t)]}{\sqrt{Var[TAIC(t)]}} > 0\right\} \approx 0.5 \tag{17}$$

In this case, the net premium principle is considered inadequate. The main reason lies in the inability of this principle to ensure the financial stability of the insurance company, particularly with respect to the unacceptable level of bankruptcy risk.

The rationale for introducing a risk premium lies in the necessity to meet the insurer's financial stability requirements. Specifically, the total premium charged by the company should be sufficient to cover incurred claims with a probability that approaches 1.

Based on the standard deviation principle, the insurance premium is determined according to the following expression:

$$TAIP[TAIC(t)] = E[TAIC(t)] + con \cdot \sqrt{Var[TAIC(t)]}. \tag{18}$$

The probability of a company's bankruptcy is calculated using the following expression:

$$PB\{TAIC(t) > TAIP(t)\} = PB\left\{\frac{TAIC(t) - E[TAIC(t)]}{\sqrt{Var[TAIC(t)]}} > con\right\} \approx 1 - F(con). \tag{19}$$

[475] Kočović et al. (2021), op. cit.

To determine a fixed level of risk (frl), reference tables are used to identify the parameter con^* such that $F\left(con^*\right) = 1 - \varepsilon$. It is assumed that this value $con = con^*$ determines the insurance premium with a loading component that ensures the probability of bankruptcy does not exceed the specified threshold. This is expressed as:

$$PB\{TAIC(t) > TAIP(t)\} \approx \varepsilon \; . \tag{20}$$

From this equation, it follows that determining the appropriate net insurance premium is an inverse problem relative to the condition of financial stability—in other words, it represents the inverse problem of avoiding bankruptcy.

The Poisson process is used to model the probability distribution of the number of expected claims (nc) given an average claim frequency (λ) during a specified period (t). The calculated claim amounts serve as key inputs in evaluating the insurer's risk exposure. By applying the Poisson distribution and expression (20), the probability of bankruptcy for the insurance company can be assessed. A hypothetical example is provided in Table 1. If, during a given period $t = 1$, the average number of claims is $\lambda = 3$, then the insurer expects the fixed risk level to be realized at a claim count of $nc = 6$. In such a case, the insurance premium, including the load factor that ensures the probability of bankruptcy defined by expression (20), is calculated using the following formula:

$$PB\{TAIC(t) > TAIP(t)\} = 0.05041.$$

3. METHOD FOR CALCULATING BANKRUPTCY RISKS OF INSURANCE COMPANIES

It is assumed that there is an unknown number of insurance contracts $NIC(t)$ in a given time period (t), under which claim payments are made. Accordingly, two fundamentally different modelling approaches are considered: a static model and a dynamic model. In the static approach, claim payments are made at the end of the contract period and are independent of time (t). This corresponds to modelling with an integer-valued random variable NIC that remains fixed throughout the period.

In contrast, under dynamic dynamic approach $NIC(t)$ represent a random process in which claim payments are made during the contract period (t). Each claim amount is positive and independent of the total number of contracts. The exact timing of claim events is unknown—a feature that reflects actual conditions in the insurance market. This approach allows for greater flexibility and responsiveness in risk management.

To assess the company's bankruptcy risk, the following expression is applied:

$$TAIC(t) = \sum_{i=1}^{NIC(t)} AIC_i(t). \tag{21}$$

Claims are modelled by a Poisson process with intensity λ. Claim amounts $AIC_i(t)$ are independent and identically distributed random variables, independent of the number of claims. Insurance premiums accumulated up to a certain point in time (t), denoted by $TAIP(t) = \sum_{i=1}^{NIC(t)} IP_i$ are a linear function of time, as previously defined in expression (5). The risk process $TAIC(t) = \sum_{i=1}^{NIC(t)} AIC_i(t)$, in this case corresponds to a compound Poisson process. For further details on compound Poisson processes, see Finan (2017)[476] and Challa (2012).

The insurer receives premiums from policyholders at an intensity (cr), where (cr) represents the positive premium cash flow rate. The initial capital is denoted by IIC. In the risk process defined by expression (21), the number of insurance contracts and the amount of damage per claim are treated as independent random variables. The expected value of claim amounts in period (t), denoted by $acoc_i$ is defined as:

$$\mu[TAIC(t)] = \lambda \cdot t \cdot acoc_i. \tag{22}$$

Since premiums accumulate over time, $TAIP(t) = cr \cdot t$ is a linear function of time and the arithmetic mean of the total claims in the period (t), given by the expression (22), serves as the basis for determining the load factor (fl).

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[476] Finan, M. A. (2017). An introductory guide in the construction of actuarial models: a preparation for the actuarial exam. *Actuarial Exam C/4*. Russellville, AR: Arkansas Tech University, http://faculty.atu.edu/mfinan/actuarieshall/CGUIDE.pdf (accessed 11.2.2018).

$$fl = \frac{TAIP(t)}{\mu[TAIC(t)]} = \frac{cr \cdot t}{\lambda \cdot t \cdot acoc_i}, \quad (23)$$

$$fl = \frac{cr \cdot t}{\lambda \cdot t \cdot acoc_i}. \quad (23a)$$

From equation (23a), the premium cash flow rate, or insurance premium rate, is expressed as:

$$cr = \lambda \cdot acoc_i \cdot fl. \quad (24)$$

The load factor given by (23a) represents the ratio between the excess premium rate and the claim payment rate for insured events.

The determination of the load factor based on empirical data and the calculation of the correction coefficient depend on the insurer's initial capital, as incorporated into expression (24). Based on this, both the lower and upper bounds for the probability of bankruptcy can be established. As an illustrative example, base values are assumed for the average number of claims λ in the period t. The load factor is then calculated according to the expected number of claims and the associated Poisson probability distribution.

In the illustrative example, it is assumed that the average claim amount is 5,000 RSD. The assumption is that the insurance company opts for a safety load of 20%. The safety load accounts for uncertainty in risk assessment, administrative costs, and the insurer's profit margin. The procedure for determining the load factor is based on the following steps:

- Base net premium = average number of claims * average claim amount
- Base gross premium = base net premium * (1 + safety load)
- Gross premium for expected number of claims = base gross premium + expected number of claims * (1 + safety load) * Poisson probability
- Load factor = gross premium for the expected number of claims / base gross premium

Table 2. Assumed values for a hypothetical example

Average number of claims λ=3 in period (t)	
Average claim amount	5,000.00
Safety load (Load factor)	20%

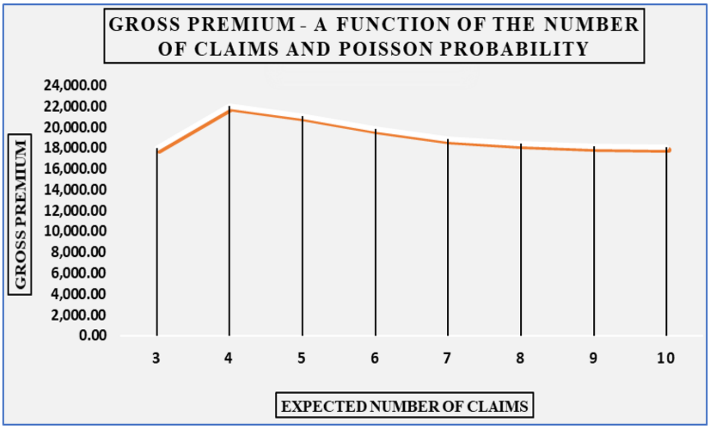
Source: Author's assumptions

Table 3. Calculated results for a hypothetical example

Expected number of claims	Net premium	Premium with safety load	Poisson's probabilities	Gross premium	Load factor	Load factor (%)
3	**15,000.00**	**18,000.00**		**18,000.00**		**0.00%**
4			0.16803136	22,032.75	1.2240	22.40%
5			0.10081881	21,024.56	1.1680	16.80%
6			0.05040941	19,814.74	1.1008	10.08%
7			0.02160403	18,907.37	1.0504	5.04%
8			0.00810151	18,388.87	1.0216	2.16%
9			0.0027005	18,145.83	1.0081	0.81%
10			0.00081015	18,048.61	1.0027	0.27%

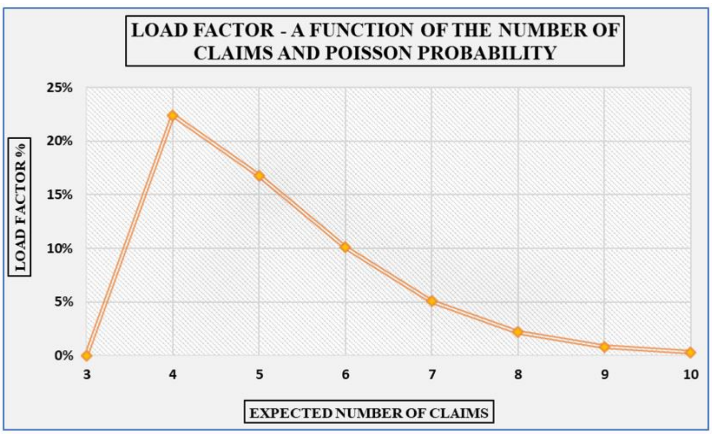
Source: Author's calculation

Figure 2. Gross premiums by expected number of claims



Source: Author's illustration

Figure 3. Load factors by expected number of claims



Source: Author's illustration

Based on the initial assumptions—average number of claims $\lambda = 3$, average claim amount of 5,000 RSD and a safety load of 20%, the following conclusions are drawn:

- As the expected number of claims increases relative to the average claim amount, the Poisson probability of incurred claims decreases and tends toward zero. Consequently, the gross premium approaches the base gross premium;
- Likewise, the load factor gradually converges toward the safety load value. This confirms that the chosen safety load is sufficient to maintain the insurer's liquidity.

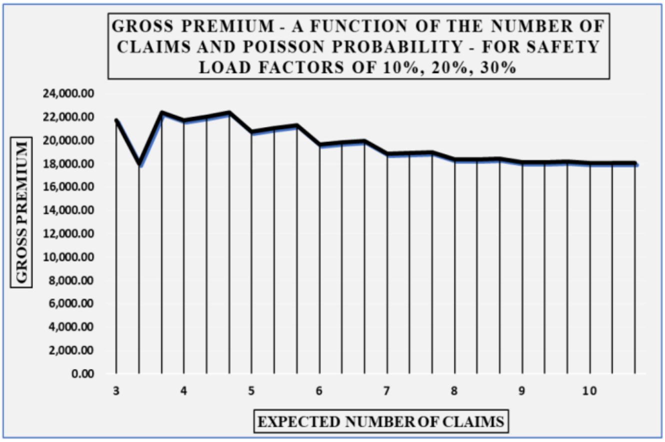
A sensitivity test was performed by varying the safety load at 10%, 20%, and 30%, while keeping all other parameters constant.

Table 4. Calculated results with varying safety loads

Expected number of claims	**Safety load**	**Net pemium**	**Premium with safety load**	**Poisson's probab.**	**Gross premium**	**Load Factor**	**Load Factor (%)**
3	10%	15000	16,500.00	0.22404	21,696.6	1.2053	20.54%
	20%	**15000**	**18,000.00**	**0.22404**	18,000.0	1	0.00%
	30%	15000	19,500.00	0.22404	22,368.8	1.2427	24.27%
4	10%	20000	22,000.00	0.16803	21,696.6	1.2053	20.54%
	20%	20000	24,000.00	0.16803	22,032.7	1.2240	22.40%
	30%	20000	26,000.00	0.16803	22,368.8	1.2427	24.27%
5	10%	25000	27,500.00	0.10081	20,772.5	1.1540	15.40%
	20%	25000	30,000.00	0.10081	21,024.5	1.1680	16.80%
	30%	25000	32,500.00	0.10081	21,276.6	1.1820	18.20%
6	10%	30000	33,000.00	0.05040	19,663.5	1.0924	9.24%
	20%	30000	36,000.00	0.05040	19,814.7	1.1008	10.08%
	30%	30000	39,000.00	0.05040	19,965.9	1.1092	10.92%
7	10%	35000	38,500.00	0.02160	18,831.7	1.0462	4.62%
	20%	35000	42,000.00	0.02160	18,907.3	1.0504	5.04%
	30%	35000	45,500.00	0.02160	18,982.9	1.0546	5.46%
8	10%	40000	44,000.00	0.00810	18,356.4	1.0198	1.98%
	20%	40000	48,000.00	0.00810	18,388.8	1.0216	2.16%
	30%	40000	52,000.00	0.00810	18,421.2	1.0234	2.34%
9	10%	45000	49,500.00	0.00270	18,133.6	1.0074	0.74%
	20%	45000	54,000.00	0.00270	18,145.8	1.0081	0.81%
	30%	45000	58,500.00	0.00270	18,157.9	1.0087	0.88%
10	10%	50000	55,000.00	0.00081	18,044.5	1.0024	0.25%
	20%	50000	60,000.00	0.00081	18,048.6	1.0027	0.27%
	30%	50000	65,000.00	0.00081	18,052.6	1.0029	0.00%

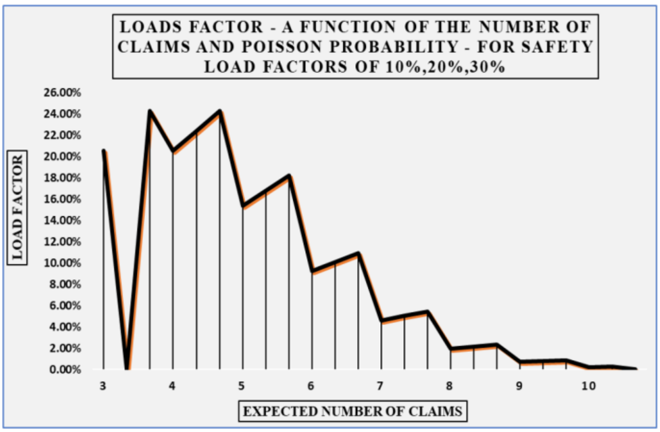
Source: Author's calculation

Figure 4. Gross premiums with different safety loads



Source: Author's illustration

Figure 5. Load factors with different safety loads



Source: Author's illustration

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Based on the results obtained for the application of a load factor of 10% or 30%, under the assumed base conditions - the average number of claims λ, the average amount of claims of 5,000 RSD and the safety load of 20%, the following conclusions are drawn:

- For $\lambda = 3$, changing the safety load to 10% or 30% results in load factor values that differ from the baseline (20%) by between 0.54% and 4.27%.

- For $\lambda = 4$, a safety load of 20% increases the load factor by 2.4%. Changes to 10% or 30% again yield a variation in load factor within the range of 0.54% to 4.27%.
- As the expected number of claims increases, the load factor converges toward the safety load, confirming that the selected safety load ensures the insurer's liquidity.
- With an average of 3 claims and a base gross premium of 18,000 RSD, changing the safety load from 10% to 30% increases gross premium amounts by RSD 3,696.69 to RSD 4,368.82.
- For an expected number of 4 claims, the gross premium increases by RSD 4,032.75 under a 20% safety load. Adjustments to 10% or 30% change the gross premium within the same interval (RSD 3,696.69 to RSD 4,368.82).
- An increase in the expected number of claims leads to a decrease in the gross premium value. With further increases, the gross premium approaches the base gross premium.

The research conclusions are derived from the development of a dynamic process-based model. Specifically, the model analyses the relationship between premium inflows from concluded insurance contracts and claim outflows related to all contracts entered into by the insurer on the insurance market. Within this framework, the model primarily evaluates the insurer's financial stability over a defined time horizon.

Based on the developed model, it is concluded that a key determinant of insurers' financial stability is the correlation between received insurance premiums and claims paid for insured events. This correlation reflects several underlying factors: the rate of premium collection, the frequency and timing of claim payments, and the statistical distribution of incurred claims.

The model's key limitations arise from how financial stability is defined. These conditions are considered sufficient when collected premiums are equal to the funds available for claim settlements during the observed period. Additional limitations relate to the current treatment of risks across different insurance contracts. A more comprehensive analysis of diverse risk types is essential for improving model accuracy and reinforcing financial resilience. Future research should aim to include multiple, or ideally all, types of risks in assessing the financial stability of insurance companies operating in the market