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Cryptography in Organizing Online Collaborative Math Problem Solving

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Abstract: The aim of this study is to examine the potential of cryptographic techniques in enhancing the organization of online group work for solving mathematical problems, while applying differentiated instruction. Engaging students in mathematics often requires additional motivational strategies and compelling incentives for sustained effort. Online group work presents a valuable opportunity for collaboration and intensive communication in solving mathematical problems. However, it also poses challenges, particularly concerning academic integrity and the risk of unauthorized copying. To address these issues, this study proposes the integration of cryptographic protocols with differentiated instruction in online collaborative tasks. Specifically, various levels of problem-solving assistance are made accessible only when the majority of the group members reach a consensus. Assistance is unlocked through the submission of individual cryptographic key segments, assigned by the instructor. A group password-required to access incremental guidance-can be generated only when a sufficient number of key segments have been submitted. This mechanism facilitates progress monitoring and fosters group accountability. The paper illustrates this approach with an example from mathematics instruction, supported by a Python-based software tool designed to aid collaborative learning. The software employs Lagrange interpolation to generate unique key parts for each participant. The method was piloted with six pre-service teachers in Serbia, and the qualitative findings are discussed alongside implications for educational research and practice.

Keywords: *collaborative learning, cryptographic techniques, mathematics differentiated instruction, online group work, problem-solving strategies.*

Introduction

The Internet and digital technologies have profoundly transformed education over the past decade, reshaping the landscape of distance and online learning (Adedoyin and Soykan, 2023). Virtual learning environments offer numerous advantages, such as flexibility, international collaborations, enriched educational experiences, increased student engagement, and the potential for anonymity (Allen et al., 2002). These benefits, combined with improved faculty development and more comprehensive feedback mechanisms, have underscored the growing importance of online education in contemporary pedagogy (Appana, 2008). Nevertheless, significant challenges remain—particularly in maintaining student motivation and preventing disengagement in virtual learning settings (Lee, 2010).

One area where innovation may address these challenges is in online mathematics education. The integration of cryptography within this domain holds promise, as it may increase student motivation by introducing an element of intrigue and intellectual challenge (Koblitz, 1997). Although cryptography is traditionally associated with securing data, its educational potential—especially in fostering collaborative problem-solving—has been largely overlooked. This paper seeks to bridge that gap by proposing a novel approach to organizing online collaborative math problem-solving through the use of cryptographic techniques.

Collaborative problem-solving (CPS) plays a crucial role in mathematics education, fostering essential 21st-century skills and enhancing student engagement (Felmer, 2023). By working together to tackle complex problems, students develop critical thinking, communication, and teamwork abilities, all of which are vital in today's interconnected world. This approach not only deepens their mathematical understanding

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The COVID-19 pandemic accelerated digital transformation in education, compelling institutions to rapidly adopt online platforms and practices that otherwise might have taken years to implement (Adedoyin and Soykan, 2023). This transition highlighted the potential of collaborative learning in virtual settings, offering both academic and social benefits to students (Đorđević and Milutinović, 2021). Research suggests that students working collaboratively often outperform individuals working alone, as shared goals promote accountability and active participation (Johnson and Johnson, 1999). Cooperative learning also supports students' social and psychological needs, fostering interaction and promoting self-directed learning (Slavin, 1995).

Within the context of online math education, group work has emerged as an effective strategy for developing key skills. According to the ISTE standards (International Society for Technology in Education, 2016), students should engage with digital tools to enhance problem-solving, decision-making, and global collaboration. However, the effectiveness of online group work is frequently hampered by issues such as academic dishonesty, intra-group conflict, and unequal participation (Vienović and Adamović, 2013). Cryptography offers a potential solution to these issues by enabling secure, consensus-based collaboration. By assigning unique cryptographic keys to each participant and requiring group agreement to unlock portions of problem-solving tasks, the model can mitigate cheating and foster cooperation.

This paper introduces and explores a cryptographic model designed to structure and support online collaborative mathematics problem-solving. It examines the model's potential to enhance motivation, ensure academic integrity, and promote meaningful collaboration, thereby addressing a critical gap in the existing literature (Đorđević and Milutinović, 2021).

Research Objective and Question

This study aims to evaluate the effectiveness of a cryptographic model in facilitating collaborative problem-solving and differentiated instruction among pre-service teachers. Specifically, the research seeks to answer the following question:

- How does the integration of cryptographic techniques in online collaborative math problem-solving with differentiated instruction influence pre-service teachers' engagement, mathematical understanding, and collaboration?

Scope and Significance

This study focuses on the design, implementation, and evaluation of a cryptographic model for online collaborative mathematical problem-solving, embedded within a differentiated instruction framework for pre-service teachers. The model assigns unique cryptographic keys to each participant, requiring group consensus to unlock progressive levels of problem-solving support. The significance of this research lies in its potential to:

- Enhance student motivation by introducing differentiated instruction elements into the learning process;
- Protect academic integrity by mitigating issues such as answer copying and uneven participation;
- Foster effective collaboration and the development of 21st-century skills, including critical thinking, communication, and teamwork.

By addressing these aspects, the study contributes to the growing body of knowledge on integrating cryptography into educational practices, offering insights into innovative methods for engaging students and promoting deeper learning in online environments.

Literature review

Online education

Online education refers to the process of delivering educational content through Internet-based platforms (Lee, 2010). Owing to the flexibility and convenience of such courses, research has highlighted their potential and positive impact on student learning outcomes (Allen et al., 2002). Online learning encompasses a variety of computer-based tools, including multimedia resources, simulations, and educational games across diverse subject areas (Keengwe and Kidd, 2010). Beyond content delivery, it aims to foster metacognitive, reflective, and collaborative skills. Self-directed learning and autonomy in managing learning experiences also play a vital role in online environments, contributing to improved academic performance (Keengwe and Kidd, 2010). Mandić et al. (2018) suggested that modular teaching systems, integrating face-to-face, web-based, and supervised instructional methods, promote independent learning, foster curiosity, and encourage active student engagement, contributing to intellectual growth. This is a form of blended or hybrid learning that incorporates elements of both traditional and online learning.

In higher education, online environments are typically characterized by active learning and student-centered strategies (Barker, 2003). However, maintaining faculty engagement is a key challenge for institutions offering online learning. Many educators are hesitant to transition from traditional courses to online formats, often citing a lack of institutional support, training, and resources (Keengwe et al., 2009). Faculty engagement is further influenced by factors such as perceived usefulness, ease of use, and digital competencies (Milutinović, 2020).

Obstacles to online learning adoption include faculty workload, technical issues, lack of administrative backing, and concerns regarding course quality (Nelson and Thompson, 2005). Additionally, faculty members often face expanded roles in online settings, taking on responsibilities as mentors, coaches, and counselors (Gratz and Looney, 2020). To address these challenges, institutions should provide robust technical support and allocate dedicated time for faculty to develop and manage online courses. Faculty must adapt to new roles, assuming responsibilities as facilitators, designers, and technologists in online learning environments (Panda and Mishra, 2007). Without institutional support, educators face increased pressure to manage these roles independently, further complicating online instruction (Williams, 2003). Therefore, education institutions should provide streamlined technical solutions to support online group work, thereby reducing faculty workload and enhancing instructional efficiency.

Collaborative learning and problem-solving

Collaborative learning is an instructional approach in whichindividuals work jointly, sharing responsibility and authority to achieve common objectives (Johnson and Johnson, 1999). It emphasizes consensus-building over competition, promoting social skills and teamwork among students (Cohen and Cohen, 1991). Extended group collaboration encourages community building, as students often stay connected beyond classroom activities (Bean, 1996). This interaction enhances social support networks, allowing students to better understand differences and work through social challenges (Cohen and Willis, 1985).

Sherman (1991) highlights that collaborative learning provides effective environments for conflict resolution, fostering social strategies to address disputes (Johnson and Johnson, 1990). Collaboration helps students develop responsibility for their peers, leading to stronger interpersonal bonds (Bonoma et al., 1974). Collaborative learning also enhances cognitive skills, as students are actively engaged in discussing and problem-solving together (Webb, 1980). The structure of collaboration allows teachers to assess students' thinking processes, offering opportunities for further support (Peterson and Swing, 1985).

Collaborative learning environments reduce student anxiety, especially in unfamiliar settings, fostering better engagement and motivation (Kessler et al., 1985). The benefits of collaboration are well-documented, leading to improved academic outcomes, stronger relationships, and enhanced social and psychological well-being (Laal and Ghodsi, 2012).

Tian and Zheng (2024) suggest employing online collaborative problem-solving (CPS) techniques to enhance students' cognitive and emotional learning outcomes. To further improve students' social learning performance, they advocate for instructors to thoughtfully design collaborative scaffolding that actively engages students in purposeful and constructive online CPS activities.

Collaborative Problem-Solving (CPS) is underpinned by several key theoretical frameworks that emphasize the importance of social interaction and active engagement in learning. Lev Vygotsky's Social Constructivism posits that knowledge is constructed through social interactions, highlighting concepts such as the Zone of Proximal Development (ZPD) and scaffolding (Vygotsky, 1978). The ZPD represents the gap between what a learner can do independently and what they can achieve with guidance from more knowledgeable others. Scaffolding refers to the support provided to learners that enables them to perform tasks they cannot complete alone, fostering cognitive development through collaborative efforts.

Additionally, Piaget's theory of individual constructivism contributes to understanding CPS by focusing on cognitive development through peer interactions. His work suggests that peer cooperation is a significant social relation that supports cognitive growth, especially when peers with different perspectives engage in problem-solving together. This interaction can lead to cognitive conflict, which is essential for learning and development (Baucal et al., 2023).

Furthermore, Problem-Based Learning (PBL) shares foundational principles with CPS, emphasizing experiential and situated learning. PBL is grounded in theories of experiential learning, contextualized learning, collaborative learning, and self-regulated learning (Chen, 2022). It posits that learning is most effective when it occurs within the context in which knowledge will be applied, encouraging learners to take responsibility for their learning processes through active problem-solving.

Online collaborative group work

Historically, collaborative learning was restricted to in-person settings due to logistical constraints, such as finding a common time and place for students to meet (Kimball, 2002). However, the rise of Internet-based communication has expanded the possibilities for online collaboration (Collis, 1996). Learning management systems like Moodle and Google Classroom provide tools for synchronous and asynchronous discussion, enabling remote group work (Paloff and Pratt, 1999). These platforms have enhanced peer interaction, even among distance learners who had limited opportunities for collaboration before (Piezon and Ferree, 2008).

Online group work, however, presents challenges that often hinder student participation. Concerns about unequal contributions, managing group members' expectations, and the reduced flexibility of collaborative schedules contribute to student reluctance (Brindley et al., 2009). While technology enables collaboration, it also imposes constraints that can make students apprehensive about group work. Milutinović (2024a) found that positive attitudes toward collaboration improved both perceived usefulness and enjoyment of programming among primary school students in Serbia. This suggests that a positive attitude toward collaboration enhances students' perception of educational value and increases their enjoyment of the learning process, underscoring the importance of cultivating positive experiences in collaborative work.

Differentiated instruction in mathematics

Vygotsky's concept of the Zone of Proximal Development (ZPD) highlights the range of tasks that learners can perform with appropriate guidance. Differentiated instruction applies this by tailoring support to help students progress within their ZPD, facilitating effective learning experiences. Tomlinson's Differentiated Instruction framework (Tomlinson, 2005) provides a structured approach to differentiation, focusing on modifying content, process, product, and learning environment based on students' readiness levels, interests, and learning profiles. This model serves as a practical guide for implementing differentiated strategies in mathematics education to address student diversity effectively (Kurnila and Juniati, 2025). Differentiated instruction in mathematics is an inclusive approach that tailors teaching strategies to meet diverse student needs (Gervasoni and Lindenskov, 2010). Hackenberg et al. (2021) defined differentiated instruction as the proactive adaptation of teaching strategies to align with students' mathematical thinking while fostering a unified classroom community. Through analysis of 10 episodes across experiments, they identified five teaching practices that facilitate this approach: utilizing research-based insights into students' mathematical reasoning, offering purposeful choices and multiple pathways, engaging responsively during group activities, monitoring small group dynamics, and leading whole-class discussions that integrate diverse perspectives.

Research in mathematics education has highlighted the multifaceted nature of differentiation, emphasizing its importance in curriculum design, assessment, remote learning environments, teacher knowledge, and inclusive practices. Saxe et al. (2013) conducted a study where fourth-grade students received differentiated instruction using number lines to learn fractions and integers, resulting in significantly greater learning gains compared to those in standard classrooms. Similarly, Chamberlin and Powers (2010) observed that prospective teachers in differentiated mathematics courses exhibited statistically significant improvements from pretest to posttest compared to their counterparts in traditional classes.

Effective differentiation is increasingly critical in creating inclusive classrooms that accommodate students with varying academic abilities. As educational expectations evolve, teachers are required to implement pedagogies that foster success for all learners, including those with low achievement levels (Shernoff et al., 2011). Computerized systems and differentiation as part of a broader educational context positively impact students' language and math performance in primary education (Deunk et al., 2018).

Despite increased awareness of differentiated instruction, teachers often struggle to implement these approaches in practice (Bobis et al., 2019). Teachers have reported gaining a better understanding of differentiation through targeted professional learning and enriched curricula, although barriers to effective implementation persist (Hayden et al., 2023). Research suggests that further support is needed to overcome the constraints limiting teachers' ability to differentiate effectively.

Cryptography in organizing online collaborative group work

Cryptography, derived from the Greek words *kryptós* (hidden, secret) and *graphein* (to write), plays a crucial role in securing data (Dooley, 2008). Modern cryptography systems, including symmetric (Data Encryption Standard – DES, and Advanced Encryption Standard – AES), and asymmetric (Rivest-Shamir-Adleman – RSA) encryption, are widely used to protect privacy and secure communications (Vienović and Adamović, 2013). In education, cryptography can be leveraged to organize online collaborative group work.

A cryptographic system based on Galois fields has potential applications for structuring online collaboration. Đorđević and Milutinović (2021) proposed a model using the Galois field GF (2^8), commonly used in the AES system, to secure group decision-making. The model applies the Lagrangian interpolation polynomial to generate key parts for group members. Each student is assigned a portion of the cryptographic key, and a two-thirds majority is required to reconstruct the original key.

In this model, if fewer than two-thirds of the group members participate, the key cannot be calculated, preventing unauthorized actions. The model ensures fairness in group decisions by distributing key ownership among participants. If a two-thirds majority agrees, the key can be reconstructed, allowing the group to perform the desired action. This system not only secures the decision-making process but also addresses potential issues related to uneven participation in online group work (Đorđević and Milutinović, 2021).

Materials and Methods

Aim of the Study

This study aims to evaluate the effectiveness of a cryptographic model in enhancing collaborative problem-solving among pre-service teachers. The model is designed to support differentiated instruction in online group settings by providing structured assistance levels to enhance engagement and mathematical understanding.

Research Design

A qualitative case study approach was employed to gain an in-depth understanding of how the cryptographic model influences collaborative problem-solving and differentiated instruction among pre-service teachers. This design is appropriate for exploring complex phenomena within their real-life contexts, allowing for a comprehensive examination of participants' experiences and interactions with the model.

Participants and Sampling

The study involved a purposive sample (Ahmad and Wilkins, 2024) of six pre-service teachers from the Faculty of Education in Jagodina. Purposive sampling was chosen to select individuals who possess prior knowledge of linear equations and familiarity with various solution methods, including graphing, elimination, and substitution. This criterion ensured that participants had the necessary background to engage meaningfully with the collaborative tasks and the cryptographic model.

Data Collection Tools and Procedures

Data were collected through multiple methods to ensure a rich and comprehensive understanding of the participants' experiences:

1. Observations: Participants engaged in a week-long collaborative task focusing on solving a system of linear equations. Their interactions, problem-solving strategies, and use of the cryptographic model were observed and documented.
2. Interviews: Semi-structured interviews were conducted with each participant post-intervention to

gather insights into their perceptions of the cryptographic model's effectiveness, its impact on their collaborative problem-solving abilities, and its role in supporting differentiated instruction.

3. Artifacts Analysis: Participants' work products, including solution processes and communication logs, were collected and analyzed to triangulate data from observations and interviews.

Data Analysis Techniques

Thematic analysis was employed to analyze qualitative data from observations, interviews, and artifacts. This involved coding the data to identify recurring themes and patterns related to the research objectives. The analysis focused on understanding how the cryptographic model influenced collaborative dynamics, supported differentiated instruction, and impacted participants' mathematical understanding.

Justification of Methods

The qualitative case study design was selected for its strength in exploring complex educational interventions within their natural settings (Ancker et al., 2021). Purposive sampling ensured that participants had the requisite background to engage with the study's tasks, aligning with the goal of assessing the cryptographic model's effectiveness among individuals with foundational knowledge of linear equations. The combination of observations, interviews, and artifact analysis provided a comprehensive data set, allowing for triangulation and enhancing the validity of the findings. These methods collectively align with the research goals by facilitating an in-depth exploration of the cryptographic model's impact on collaborative problem-solving and differentiated instruction among pre-service teachers.

Mathematical foundation and the development of user-friendly application

A common mathematical foundation for forming a cryptographic system is Galois fields and their extensions. The model used in this paper is based on the finite Galois field GF (2^8), which is most commonly used in AES systems (Daemen and Rijmen, 2002; Desoky and Ashikhmin, 2006; Murphy and Robshaw, 2002). In such cryptographic systems, a byte is considered as an element of a binary finite field defined by the irreducible "Rijndael" polynomial $P(x) = x^8 + x^4 + x^3 + x + 1$ (Murphy and Robshaw, 2002). This field is characterized by a large number of inverse operations, using arithmetic modulo 2^8. For example, AES uses the inverse element operation in relation to multiplication in the field GF (2^8) (Desoky and Ashikhmin, 2006). Furthermore, this field includes operations on polynomials of arbitrary degree, with coefficients ranging from 0 to 255 from the field GF (2^8) (Daemen and Rijmen, 2002). One such operation is the Lagrange interpolation formula.

The challenge of constructing a continuous function from discrete data arises frequently in mathematical analysis, especially when data manipulation requires estimates beyond the provided dataset. A commonly preferred method for addressing this issue is interpolation, where the goal is to construct an approximation function that exactly matches the values of the original, typically unknown, function at the given data points. In practical computational applications, the interpolation problem can be stated as follows: given the function values at a finite set of points, determine the function's value at an intermediate or specified argument (Hussien, 2011).

Let the function f be defined by its values $f_x = f(x_k)$ at discrete points x_k where $k = 0,1,2, \dots , n$. Without loss of generality, we assume that $a \leq x_0 \leq x_1 \leq \dots \leq x_k \leq b$. If the points x_k are taken as interpolation nodes and $\varphi_k(x) = x^k$, $(k = 0,1, \dots ,n)$ is set, we arrive at the interpolation problem for the function f using an algebraic polynomial. Let this polynomial be denoted as P_n (Milovanović, 1988), i.e.:

$$P_n = a_0 + a_1 x + \dots + a_n x^n$$

Lemma 1 (Lagrange interpolation polynomial) (Kovács and Kovács, 2005): The polynomial is unique and can be represented in the form:

$$P_n(x) = \sum_{k=0}^{n} f(x_k) L_k(x)$$

where:

$$L_k = \prod_{k,j=0, k\neq k}^{n} \frac{x - x_k}{x_j - x_k}$$

The previously discussed concepts are now applicable to cryptographic decision problems within a group setting, such as a group of six members where four parts are required to generate the key. For a given integer , that lies in the range $0 \leq a \leq 255$, random integers $r_1, r_2, \ldots, r_3$ are generated within the range of 0 to 255. For these given values, a polynomial $f(x)$ is formed over the field GF (2^8):

$$f(x) = a + r_1 x + \ldots + r_3 x^3$$

It is evident that $f(0) = a$. Subsequently, the values $f(1), \ldots, f(6)$, are calculated, where these values represent elements of the finite field GF (2^8) with operations conducted according to the rules of that field. Next, ordered pairs $p_1 = (1, f(1)), \ldots, p_6 = (6, f(6))$ are formed, and a function is implemented that, using any random selection of four out of the six pairs, calculates $f(0)$ (which equals a) through the Lagrange interpolation formula.

The above principles can be applied to a group of six individuals who must reach a decision by a two-thirds majority to execute an action protected by a key. If any one member possesses the entire key, that individual can unilaterally make the decision, similar to the case where no key exists. Conversely, if the key is divided into six parts, each assigned to an individual, it is possible for five members to agree to execute the action while the sixth key holder withholds their part, creating an unfair situation in group dynamics.

When the key is represented as an integer a in the range of 0 to 255, pairs $p_1, \ldots, p_6$ can be formed as previously described, with one pair allocated to each group member. If any four key holders decide to proceed, they can utilise the Lagrange interpolation polynomial to compute . However, if three or fewer members contribute their pairs, they cannot successfully calculate the key using the Lagrange interpolation polynomial, thus preventing the execution of the desired action. Therefore, the key can be divided into six parts, allowing for the calculation of the key with any four parts, while fewer than four parts will yield an incorrect key.

For the purpose of studying the implementation of cryptography in organizing online collaborative group work, the authors developed user-friendly software tailored for both teachers and students. The software, built using the Python programming language, was designed based on the mathematical foundations presented above and algorithms presented by Đorđević and Milutinović (2021). Milutinović (2024b) argues that Python is a versatile, high-level language supporting various programming paradigms, making it ideal for implementing different algorithms. Its extensive library is especially useful for mathematical topics such as algebra, calculus, and number theory, contributing to its widespread use today.

The interface of the developed software facilitates seamless interaction while applying the cryptographic model, ensuring that group work is securely managed and decision-making is fair and transparent.

Procedure

The method implemented in this study can be used with students at all levels of education and consists of the following steps:

Step 1 - Key Construction: The professor constructed ordered pairs based on a selected key that represents sections of the key relevant to decision-making in group work. Each participant received three ordered pairs corresponding to three different levels of help, facilitating differentiated online group problem-solving. Before accessing any level of assistance, at least two-thirds of the group must agree to use their portion of the key.

Step 2 - Differentiated Instruction: The cryptographic model is straightforward to implement for administering differentiated instruction during online group work. Assistance is divided into three levels-referred to as “first,” “second,” and “third” help-for a particular task worth 10 points (Đorđević and Milutinović, 2021). Each level of assistance is safeguarded by a unique key, divided into as many ordered pairs as there are group members.

Point System and Help Retrieval:

First Help: If students require more guidance, they may use their ordered pairs to unlock the “first help,” which provides gentle direction toward the solution. Opening this assistance results in a loss of 2 points for each group member, which is recorded.

Second Help: If the “first help” is insufficient, students can access the “second help” for more detailed instructions and completed examples related to the problem, resulting in a 5-point deduction for each group member.

Third Help: Should students still struggle, they can utilize their ordered pairs to unlock the “third help,” which offers comprehensive instructions and part of the solution, incurring an 8-point penalty. Alternatively, if students feel unprepared, they may directly access the “third help” without using the first two levels.

Assessment and Feedback: The professor reviews the last level of assistance requested by the group. Points are awarded based on the level of help accessed. For example, if the “third help” is used immediately, students lose 8 points. If the group accesses the “first” and “second helps” but does not use the “third,” they only incur a 5-point penalty, as points are not cumulative.

At the conclusion of the assignment, the authors conducted semi-structured interviews with the participants to gain insights into their experiences and perspectives regarding the collaborative problem-solving process. This qualitative approach aimed to understand how the structured assistance influenced their learning and engagement in the task. The interviews provided an opportunity for participants to articulate their thoughts, feelings, and reflections, enriching the data collected and contributing to a deeper understanding of the effectiveness of the implemented method.

Group assignment example

For their assignment, the pre-service teachers worked on solving the system of linear equations.

Assignment: For the given system of linear equations

$$-x - 2y + 14z = 8$$
$$3x - 5y - 7z = 9$$
$$4x - 2y - 3z = 24$$

find the solution (x, y, z).

Students had already mastered the basics of solving linear equations and systems of equations in their Basic Mathematics course at the Faculty. The goal of this task was to enhance their skills in solving systems of linear equations using substitution.

The group work was organized using the institution’s Learning Management System, Moodle, which enabled the formation of collaborative student groups for submitting a shared assignment. The professor could assign specific activities to designated group, ensuring that group members possessed similar mathematical competencies. Interaction within the group was facilitated through forums, wikis, and databases, with access restricted to group members. The professor also controlled whether students could view additional resources.

The group assignment in Moodle for solving the given system of linear equations is illustrated in Figure 1.

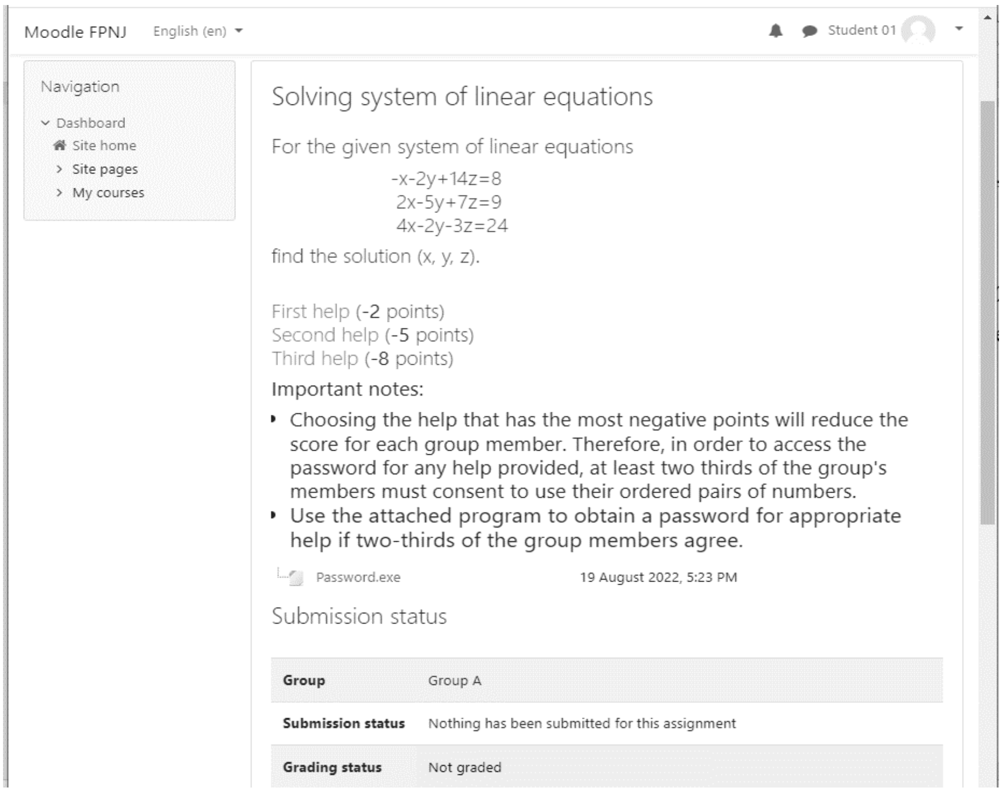


Figure 1. Moodle group assignment

Example of help retrieval

If students were unable to solve the problem collaboratively, they could decide whether to use any of the available assistance (i.e., first, second, or third help). Each group member's score would decrease if they opted for assistance, with penalties for using help. For example, to unlock the "first help," the password was set to 38, and the ordered pairs assigned to students were: (3, 226), (4, 124), (1, 212), (5, 129), (2, 156), and (6, 210), with the password, which was known known only to the teacher. Since selecting assistance would result in a decline in each group member's score, it was necessary for at least two-thirds of the group members to agree before using their ordered pairs to decipher the password for any assistance. Once two-thirds of the group reached consensus, they utilized the user-friendly software developed for this study (see Figure 2) to obtain the correct password for the selected help.

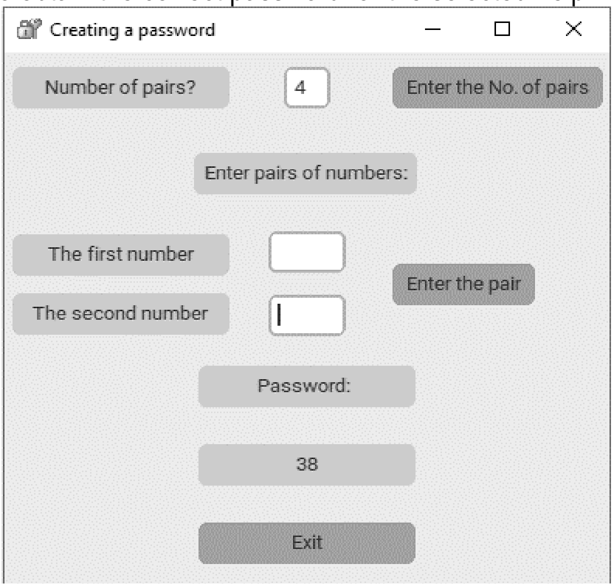
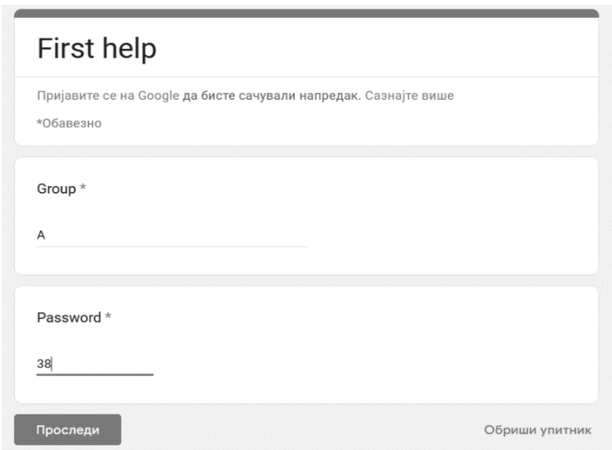


Figure 2. The application's user interface for creating passwords

After obtaining the correct password, students accessed a Google Form to input the password (see Figure 3a) and retrieve the assistance (see Figure 3b) provided in a Google Document (see Figure 4).

a)



b)

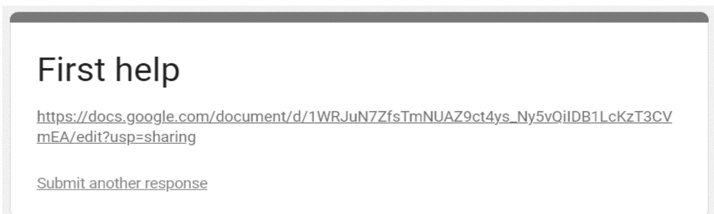


Figure 3. Google form for opening the first help a) Google form for imputing the password; b) If the password is correct, the link for the First help is provided

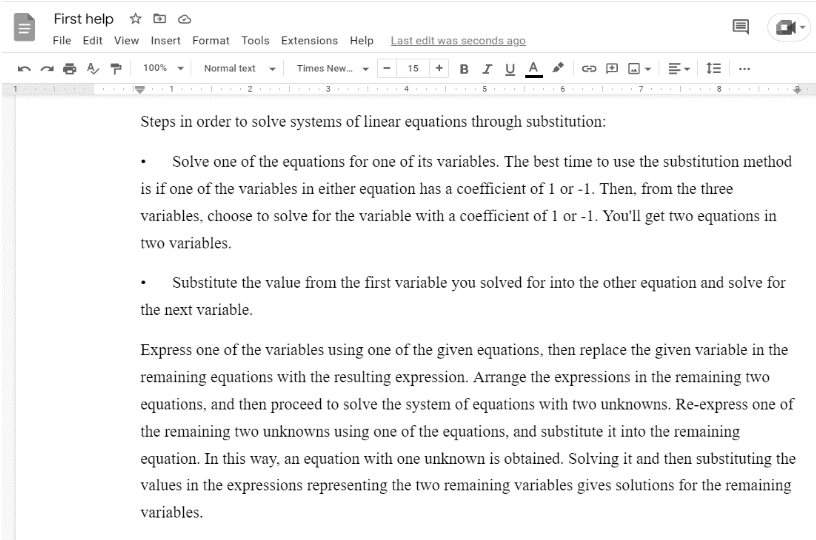


Figure 4. Protected Google document with first help

The password for "second help" was *53*, and again was known only to the teacher. Each of the students got one of the ordered pairs: *(2, 196), (3, 13), (1, 13), (4, 188), (6, 150), (5, 152)*

"Second help"

- Solve one of the equations for one of its variables. The best time to use the substitution method is if one of the variables in either equation has a coefficient of 1 or -1. Then, from the three variables, choose to solve for the variable with a coefficient of 1 or -1. You'll get two equations in two variables.
- Substitute the value from the first variable you solved for into the other equation and solve for the next variable.
- Repeat the same procedure with the two new equations.

$$-x - 2y + 14z = 8 \Rightarrow -x = 8 + 2y - 14z \Rightarrow x = 14z - 2y - 8$$
$$2x - 5y + 7z = 9$$
$$4x - 2y - 3z = 24$$

$$2(14z - 2y - 8) - 5y + 7z = 9$$
$$4(14z - 2y - 8) - 2y - 3z = 24$$

The "third help" password was *128*, known only to the teacher, and the students again receive one of the following ordered pairs: *(3, 211), (1, 117), (2, 64), (6, 189), (5, 214), (4, 106),*

"*Third help*"

- Solve one of the equations for one of its variables. The best time to use the substitution method is if one of the variables in either equation has a coefficient of 1 or -1. Then, from the three variables, choose to solve for the variable with a coefficient of 1 or -1. You'll get two equations in two variables.
- Substitute the value from the first variable you solved for into the other equation and solve for the next variable.
- Repeat the same procedure with the two new equations.
- Substitute the value from the two variables that you solved and plug it into the remaining equation and solve for the last remaining variable. This step should allow you to solve for a real number.
- After solving for the final variable, plug in the value of the most recent variable that you found into the answer of another equations with variables remaining. Note: Preferably, plug in the value to the most simplified equation.
- Therefore, you will have successfully found the answers to a system of linear equations in three variables.
- Note**.** It's always a good idea to check the solution back in the original equations just to be sure.

$$-x - 2y + 14z = 8 \Rightarrow -x = 8 + 2y - 14z \Rightarrow x = 14z - 2y - 8$$
$$2x - 5y + 7z = 9$$
$$4x - 2y - 3z = 24$$

$$2(14z - 2y - 8) - 5y + 7z = 9$$
$$4(14z - 2y - 8) - 2y - 3z = 24$$

$$35z - 9y = 25 \Rightarrow -9y = 25 - 35z \Rightarrow y = \frac{35z - 25}{9}$$
$$53z - 10y = 56$$

$$53z - 10\frac{35z - 25}{9} = 56$$

This method allowed pre-service teachers to independently decide if and how much help they needed. This had the potential to help students overcome their reluctance to actively participate in online group work. The teacher maintained a database linked to Google Forms to monitor which group members utilized the assistance, facilitating the grading process based on group participation.

Materials and methods are the second section of an IMRAD paper. Its purpose is to describe the experiment in such retail that a competent colleague could repeat the experiment and obtain the some or equivalent results. Provide sufficient detail to allow the work to be reproduced. Methods already published should be indicated by a reference: only relevant modifications should be described.

Results

The qualitative analysis of data collected from the six pre-service teachers provided valuable insights into their experiences and perceptions of using the cryptographic model for differentiated online group problem-solving.

Group Dynamics and Collaboration: All participants (6 of 6) reported an increase in collaborative efforts. The requirement for at least two-thirds of the group to agree before accessing assistance fostered a sense of responsibility and teamwork. Four participants indicated that discussions were more focused, with group members actively engaging in problem-solving rather than relying on individual efforts. However, two participants expressed frustration with delays caused by waiting for group consensus.

Utilization of Help Levels: The group opened two levels of assistance. Most participants (4 of 6) preferred to explore all possible problem-solving strategies independently before turning to assistance, while two preferred quicker access to group help. The availability of differentiated help was positively viewed as it promoted ownership of learning. On the other hand, one participant felt that the points reduction system added pressure, which somewhat hindered their willingness to use help early.

Perceived Effectiveness of the Software: The software for password retrieval was seen positively by most participants (5 of 6), who appreciated its simplicity and accessibility. However, two participants suggested adding features, such as immediate feedback on their decisions to use assistance. One participant also noted that while the system worked well, the process of entering individual key parts could be made more streamlined.

Learning Outcomes: All six participants successfully completed the mathematical task. Five participants felt more confident in their problem-solving abilities due to the collaborative structure and help system. However, one participant mentioned that they would have preferred more personalized guidance, as the group-based help did not fully address their specific learning needs.

Discussions

The research question in this study was how the integration of a cryptographic model for differentiated online group problem-solving influences collaboration, engagement, and learning outcomes among pre-service teachers. The results indicate that this model significantly contributes to increased collaboration among participants, with the majority of pre-service teachers reporting greater engagement in collective problem-solving. The help system, which provides differentiated support, was generally positively evaluated. The implementation of the cryptographic model among pre-service teachers reveals several critical insights into collaborative problem-solving in online educational settings.

The requirement for group consensus before accessing assistance appears to promote accountability and peer support, aligning with Vygotsky's (1978) social constructivist theory, which emphasizes the importance of social interaction in learning processes. This finding is consistent with studies indicating that collaborative problem-solving enhances student engagement and learning outcomes (Tian and Zheng, 2024). However, the frustration expressed by some participants regarding delays suggests a need to balance collaborative requirements with individual pacing, echoing challenges noted in collaborative learning environments (Ying and Tiemann, 2024).

The tiered assistance approach provided by the cryptographic model aligns with differentiated instruction principles, allowing students to engage at their own levels while still benefiting from group support (Tomlinson, 2001). The success of the participants in completing the mathematical task suggests that such an approach may effectively address diverse learning needs in educational contexts. The preference for independent exploration before seeking help underscores the model's effectiveness in promoting learner autonomy. Nevertheless, the pressure induced by the points reduction system highlights the delicate balance required in designing motivational elements within educational tools, as extrinsic motivators can sometimes undermine intrinsic motivation (Baucal et al., 2023).

The positive reception of the software's simplicity and accessibility aligns with research emphasizing the importance of usability in educational tools, as technology can either facilitate or hinder collaborative efforts depending on its design (Nielsen, 1993). The suggestions for immediate feedback and streamlined processes reflect a broader demand for user-centered design in educational technology, which is crucial for maintaining engagement and effectiveness.

The successful completion of the mathematical task by all participants indicates the potential effectiveness of the cryptographic model in supporting collaborative problem-solving in mathematics (Felmer, 2023; Tian and Zheng, 2024; Ying and Tiemann, 2024). The increased confidence reported by most participants is a positive outcome; however, the desire for more personalized guidance points to the ongoing challenge of addressing individual learning needs within group settings. This finding resonates with the principles of differentiated instruction, which advocate for tailoring educational experiences to meet diverse student needs (Kurnila and Juniati, 2025).

Based on these findings, it can be concluded that the cryptographic model enhances group dynamics, responsibility, and problem-solving abilities.

Implications for research and practice

The cryptographic model for differentiated online group problem-solving presents several key implications for both research and educational practice.

Its structured assistance system promotes collaboration by encouraging group consensus and accountability, fostering a more interactive and cooperative learning environment. By requiring a majority agreement to access assistance, this approach reduces the need for constant teacher oversight, shifting responsibility to students and empowering them to manage their own learning (Baucal et al., 2023). In the future, this system could evolve to integrate more flexible, student-centered strategies, enabling even greater autonomy and deeper engagement in collaborative learning. It may also support the development of lifelong learning habits, where students continuously refine their skills in a cooperative and self-regulated environment.

The model is adaptable across various subjects and educational levels, making it highly versatile and relevant in diverse curricula. It also supports differentiated instruction by ensuring that students engage meaningfully with tasks before seeking help, potentially minimizing superficial learning. Furthermore, its flexibility allows for the accommodation of diverse learning styles and paces, fostering an environment where students can take ownership of their learning. This adaptability also enables educators to tailor their teaching approaches, enhancing the overall effectiveness of instruction in a wide range of educational contexts (Kurnila and Juniati, 2025).The cryptographic approach helps reduce academic dishonesty, such as plagiarism or over-reliance on external resources, by controlling access to step-by-step assistance. This system is particularly valuable in problem-solving disciplines like mathematics, where gradual assistance can guide students without providing full solutions too early. By embedding cryptographic safeguards, the model promotes integrity in collaborative work and ensures that help is only unlocked when a consensus is reached. Additionally, it encourages students to engage with the material more deeply, promoting independent critical thinking and problem-solving skills. By ensuring that assistance is accessed in a controlled manner, the model fosters a more authentic learning experience and reinforces academic integrity in the learning process.

Incorporating artificial intelligence (AI) into this model could further enhance its effectiveness by providing adaptive support and more sophisticated monitoring of group dynamics. Likewise, integrating cryptography into AI-enhanced learning environments would add a vital layer of security and structure, especially in managing collaborative learning contexts. Cryptographic techniques could ensure that sensitive data-such as students' interactions, progress, and access to resources-are securely managed and only available under agreed-upon group conditions. This could help address growing concerns over data privacy in education, where clear policies are crucial (Mandić, 2023).

In modern education, teachers need to embrace AI as a tool that complements their expertise and enhances their ability to meet diverse student needs (Mandić, 2023; 2024). At the same time, careful planning and execution of strategies prioritizing student well-being, fairness, and effective pedagogy are essential for AI integration (Mandić et al., 2024; Milutinović and Mandić, 2022). Integrating cryptography into AI-driven learning systems could ensure secure, ethical access to assistance, upholding privacy and fairness while improving student engagement and accountability. This framework aligns with ethical concerns surrounding AI, providing a secure, transparent, and equitable model for modern education.

Conclusions

In conclusion, the implementation of the cryptographic model for differentiated online group problem-solving among six pre-service teachers has proven to be a successful and enriching experience. This method not only facilitated effective collaboration and engagement among participants but also positively influenced their learning outcomes. The findings suggest that integrating structured assistance systems into online education can enhance student motivation, accountability, and overall learning effectiveness.

In light of the ongoing challenges posed by the COVID-19 pandemic, the increasing demand for online learning, and the use of AI in education, it is crucial to establish robust organizational systems that allow students to choose when and under what circumstances they receive supportive information. Platforms like Moodle enable not only material sharing but also effective communication and project planning among group members. By implementing cryptographic techniques, group members can access appropriate resources only with the consent of the majority, ensuring that assistance is sought collaboratively. This approach fosters dialogue and exchange of ideas, making the learning process more interactive while simultaneously instilling a sense of shared responsibility for problem-solving.

Moreover, this model alleviates the instructor's burden, allowing them to focus on higher-order teaching tasks rather than micromanaging group dynamics and point deductions. It also minimizes subjectivity in assessing student contributions, fostering a fairer evaluation process. The adaptability of this method to various subjects and educational levels enhances its applicability and reduces the risk of academic dishonesty in group assignments.

This study presents a viable model for online collaborative learning across diverse educational contexts. Future research should focus on exploring the practical application of this model in mathematics education, examining students' attitudes, perceived advantages, and challenges, as well as their achievements. Additionally, investigating the model's efficacy across different subjects and contexts would further validate its potential in enhancing collaborative learning experiences.

While the method shows promise, potential limitations exist, such as students seeking external help through forums or private lessons, and technical issues like antivirus software misidentifying the program as a virus. Addressing these challenges will be essential for the effective implementation of this innovative approach in educational settings

Conflict of interests

The authors declare no conflict of interest.

Author Contributions

Conceptualization, M.V. and Đ. S.; methodology, M.V. and Đ. S.; investigation, M.V.; software, M.V. and Đ. S.; formal analysis, M.V.; writing—original draft preparation, M.V., Đ. S., and M.D.; writing—review and editing, M.V., Đ. S., and M.D. All authors have read and agreed to the published version of the manuscript.

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