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THE UNDERSTANDING OF RELATIONAL TERMS IN COMPARE-COMBINE WORD PROBLEMS ON DIFFERENT LEVELS OF EDUCATION[1]

Abstract: Routine word problems are thoroughly described and categorized to *combine*, *change,* and *compare* problems. This paper investigates how 2nd, 4th, and 6th-grade students solve integrated combine and compare problems. We used the integrated combine and compare problems with consistent language (CL) formulation, inconsistent language (IL) formulation, or more complex structure. Our research sample consists of 44 students in 2nd grade, 48 students in 4th grade, and 42 students in 6th grade from schools in Belgrade. The results show that students are more successful in solving problems with CL than with IL formulation at all levels of education. Students from the 2nd, 4th, and 6th grade are equally successful in solving the CL problem. The surprising result is the nonexistence of a significant difference in the achievement of students in 4th and 6th grade on the IL problem, which could indicate an obstacle in the development of relational term understanding after introducing algebra into mathematical education. Low achievement on the problem with more complex structure showed that students have issues with the modeling process and that they are not eager to use algebraic strategies or graphical representations. These results imply a need for a systematic approach to teaching routine problems after introducing algebra in mathematics education.

Keywords: word problems, combine problems, compare problems, problem solving strategies, mathematical education.

INTRODUCTION

There are numerous reasons why word problems have been at the center of research in mathematics education for the past few decades. One of them is in their twofold use: they could be used as *routine problems* to facilitate the devel-

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opment of the conceptual knowledge of basic arithmetic operations (Carpenter 1986; Carpenter, Hiebert, Moser 1981; Schroeder, Lester 1989), or they could be used as *non-routine* problems in the way that facilitates mathematical thinking and mathematical literacy in general (Verschaffel et al. 2010; Van Dooren et al. 2010). The categorization of routine problems with one operation to *combine*, *change*, and *compare* problems is broadly described in the literature, and every type of problem is separately investigated (Cummins et al. 1988; De Corte, Verschaffel, De Win 1985; Morales, Shute, Pellegrino 1985; Riley, Greeno, Heller 1983). Research confirmed that compare problems are the most difficult for students (e.g., Carpenter, Moser 1984; Cummins et al. 1988; Nesher, Greeno, Riley 1982; Stern 1993). In addition, compare problems are analyzed by the aspect of their language consistency. A compare problem with consistent language (CL) formulation is less challenging to the students than a compare problem with inconsistent language (IL) formulation. Even though *combine* problems are less complicated for the students than change and compare problems, there are also different types of combine problems that could be less or more challenging to the students (Riley, Greeno 1988).

It is still uninvestigated how students at different education levels solve problems created by the integration of the combine and compare problems. As we have stated, both types of problems are routine, but there are two issues that integration could bring into problem solving: 1) language consistency of the compare problem; 2) more complex structure of the combine problem. In this paper, which is part of broader research, we investigate the achievement and strategies of the students at different levels of education (2nd, 4th and 6th grade) on the integrated combine and compare problems (here referred to as *combine-compare* problems). Students at these levels of education have diverse mathematical skills. It is essential to understand obstacles that students at each level of education have in *combine-compare* problem solving and give implications for their overcoming. In their future mathematics education, solving these problems would become just a tool for solving more complex routine and non-routine problems. Hence, it is important that students can solve them correctly and efficiently.

THE CATEGORIZATION OF WORD PROBLEMS WITH ONE OPERATION

One of the most used definitions is that word problems are verbal descriptions of a problem situation in which the answer could be given by performing mathematical operations on numerical data provided in the text of the problem (Verschaffel, Depaepe, Van Dooren 2014). The above-mentioned routine word problems that could be solved with one mathematical operation (in one step) were key components in mathematics curriculums for elementary schools worldwide. They are considered the basis for learning in mathematics classrooms; hence the voluminous research in the 80s and 90s was focused on problems with one operation and their characteristics, categorization, process of solving, and impact on students' thinking.

The basic categorization of word problems with one operation (addition or subtraction) appeared at the beginning of the 80s. Numerous empirical studies with children aged five to eight showed that even if problems could be solved with the same arithmetic operation, they belong to different semantic categories, which suggests that different strategies for representing and solving problems trigger different types of mistakes (Fuson 1992). Based on the semantic structure and situation described in the text of the problem, they are classified into three categories: *combine*, *change*, and *compare* problems (Cummins et al. 1988; De Corte, Verschaffel, De Win 1985; Morales, Shute, Pellegrino 1985; Powell et al. 2009; Riley, Greeno, Heller 1983; Verschaffel 1994). The categorization served as a guideline for numerous future research studies. The other aspect of categorizing word problems is whether they describe static or dynamic situations (Carpenter, Hiebert, Moser 1981). We provide the examples by De Corte and Verschaffel (1986) that explain the difference between the three categories and the relationships (dynamic/static) in Table 1.

Table 1. Categorization of word problems – semantic structure and dynamic of the situation.

Type	Example	Situation
Change	Pete had 3 apples. Ann gave him 5 more apples. How many apples does Pete have now?	Dynamic situation – implied action in which one set is joined to another; Two entities are the subset of the third.
Combine	Pete has 3 apples. Ann has 5 apples. How many apples do they have altogether?	Static relationship; Two entities are the subset of the third.
Compare	Pete has 3 apples. Ann has 8 apples. How many apples does Ann have more than Pete?	Static relationship; One of the sets described in the problem is completely disjoint from the other two

Riley and Greeno (1988) investigated the achievement of students of different ages in solving problems in all three categories. Students were more successful in the combine word problems than in change word problems and least successful in the compare word problems. Many studies also confirm that compare problems are the biggest challenge for students (Briars, Larkin 1984; Carpenter, Moser 1984; Cummins et al. 1988; Morales, Shute, Pellegrino 1985; Nesher, Greeno, Riley 1982; Okamoto 1996; Riley, Greeno 1988; Riley, Greeno, Heller 1983; Stern 1993).

Research also deals with further analysis and categorization of *change*, *combine,* and *compare* problems (Riley, Greeno 1988).

The *change* problems are subcategorized according to whether the result, change, or start value is unknown. We will not represent the classification of change problems because they are not used in our research.

For the *combine* problems, classification is made according to the position of the unknown entity set. There are two types of these problems: problems with the unknown combination (the total number or the whole) and problems with an unknown subset (part). Some of the examples provided by Riley and Greeno (1988) that have linguistical forms like the one we used in this research are presented in Table 2.

Table 2. Categorization of combine and compare problems.

Category	Subcategory	Example
Combine problems	Combination (the total number or the whole) unknown	(1) Joe has 3 marbles. Tom has 5 marbles. How many marbles do they have altogether?
	Subset (part) unknown	(2) Joe and Tom have 8 marbles altogether. Joe has 3 marbles. How many marbles does Tom have?
Compare problems	Difference unknown	(3) Joe has 5 marbles. Tom has 8 marbles. How many marbles does Tom have more than Joe?
	Compared quantity unknown	(4) Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?
	Referent unknown	(5) Joe has 8 marbles. He has 5 more marbles than Tom. How many marbles does Tom have?

There are three types of *compare* problems: when the difference set is unknown, when the compared set is unknown, and when the referent set is unknown (Table 2). In other words, in *combine* problems, any of the entities (the difference, the compared quantity, or the referent) can be left to the students to find. Students are most frequently asked to find the unknown difference. Even if all three types of problems represent the same relationship, the most difficult for the students are the ones with unknown referents; the problems with unknown compared quantities and the problems with unknown differences seem to be the least difficult for the students (Schumacher, Fuchs 2012). One of the reasons why problems with an unknown referent are the most difficult type of compare problem is that they require an understanding of the symmetrical relationship between relations *more than* and *less than* (Stern 1993).

Another approach to classifying compare word problems is based on language formulations. Lewis and Mayer (1987) describe two types of problems: consistent language problems (CL) and inconsistent language problems (IL). In CL problems, the mathematical operation can be easily discovered using the relational term (key term, keyword). For example, if the relational term is *more than*, the task's solution is adding quantities. In contrast, in IL problems, the mathematical operation could not be found by keyword. For example, a problem contains *more than*, but it must be solved by subtraction. In Table 2, problem (4) is a CL problem and problem (5) is an IL problem. We can see that compare problems with an unknown compared quantity have CL formulation and compare problems with a referent unknown have IL formulation.

OBSTACLES IN SOLVING COMPARE WORD PROBLEMS

There are several hypotheses about the source of the difficulties in solving compare problems. Many researchers (Schumacher, Fuchs 2012; Riley, Greeno, Heller 1983; Riley, Greeno 1988; Resnick 1983; Okamoto 1996; Okamoto, Case 1996) emphasize that younger students could not understand that the difference between the number of elements of two sets could be expressed in parallel ways by using terms *more* and *less.* Younger students lack knowledge and experience with language describing quantities' relations. Hence, there is research that implies that students have to learn about the symmetry of the comparison – that sentences "Monica has 11 goats less than Martin", and "Martin has 11 goats more than Monica" can be used to describe the same situation (Okamoto 1996; Okamoto, Case 1996).

Second, the semantical relations between known and unknown quantities could be less or more explicit, which could bring difficulties in understanding the situation described in the problem (De Corte, Verschaffel, De Win 1985; De Corte, Verschaffel, Pauwels 1990; Marzocchi et al. 2002; Verschaffel, De Corte, Pauwels 1992). For successful problem solving, it is essential to understand the situation and these semantical relations (Cummins 1991; Cummins et al. 1988; Kintsch 1988; Kintsch, Greeno 1985).

The third and the most researched hypothesis about the difficulties in solving compare problems is in the consistency of the relational term (i.e., keyword – *more than* or *less than*) used in the problem and the mathematical operation needed for its solving. There are two approaches to solving compare problems (Hegarty, Mayer, Monk 1995). In the first approach, students automatically translate *more than* into addition and *less than* into subtraction and develop a solving plan that implies combining the numbers given in the problem and translated operations. This approach, which is a superficial problem-solving strategy, is related to unsuccessful problem solvers. Researchers refer to this approach differently, as: "compute first and think later" (Stigler, Lee, Stevenson 1990: 15), keyword method (Briars, Larkin 1984), and number grabbing (Littlefield, Rieser 1993).

Riley, Greeno, and Heller (1983) state that students intuitively rely on the automatically activated rule – *add if the relation is 'more' and subtract if the relation is 'less'*. In some word problems, this approach really leads to the correct solution; the numbers and keywords from the text can be translated directly into mathematical expressions, but with these problems, students only practice computing skills and imitate the problem-solving process without using conceptual understanding and logical thinking (Lithner 2008; Boesen et al. 2014). In other words, students using this approach do not construct an adequate situational and mathematical model of a problem.

The other approach, related to successful problem solvers, requires constructing a situational model and using an adequate strategy for its solving. Accordingly, many studies started investigating language consistency in the compare problems.

As we previously mentioned, Lewis and Mayer (1987) recognized two parallel types of problems: CL and IL problems. They confirmed that students generally make more mistakes on IL problems than on CL problems, especially when they choose the mathematical operation. In IL problems, they tend to choose the opposite operation. This is called the consistency effect which is investigated and confirmed in many studies (Hegarty, Mayer, Green 1992; Hegarty, Mayer, Monk 1995; Stern 1993; Verschaffel 1994; Verschaffel, De Corte, Pauwels 1992; Lewis, Mayer 1987; Hegarty, Mayer, Green 1992; Lewis 1989; Verschaffel, De Corte, Pauwels 1992; Pape 2003; Van der Schoot et al. 2009).

We can question and investigate if the consistency effect is related to students' age. Stern (1993) conducted two studies in which he investigated the students' understanding of the symmetry of terms *more* and *less* in solving compare problems with the unknown referent. In these studies, he presented pictures to first graders and asked the students to pair them with relational sentences. For example, students had to state which of the sentences were correct: "there are 2 more cows than pigs", and "there are 2 pigs less than cows". Even if students understood the meaning of the sentences, they did not understand that both relations (more and less) can be used to express the same relationship. Studies also showed that low student achievement on this task was related to their ability to solve compare problems with the unknown referent. As the studies show, one of the possible reasons for difficulties in compare problems is students' incomprehension of relational terminology. Elementary school students do not have the conceptual knowledge needed for a complete understanding of compare problems, which could explain their difficulties in solving this type of problem (Cummins et al. 1988; Riley, Greeno, Heller 1983). They do not have the ability to understand and process the meaning of the problem and recall the adequate problem structure (Koedinger, Nathan 2004).

The findings of Boonen and Jolles (2015) were different. They researched why second graders had more difficulties with compare problems than with combine and change problems. As was expected, students made more mistakes on compare problems than on the other two types, but surprisingly they did not confirm the consistency effect. The second graders in this study were equally successful in solving CL and IL problems. The explanation they provided for these results is that students generally showed difficulties processing the relational terms *more* *than* and *less than*, which could be the reason for the results not confirming the consistency effect.

Later research investigated the consistency effect on students in higher grade levels and university (Pape 2003; Van der Schoot et al. 2009). The effect is confirmed with university students (e. g. Hegarty, Mayer, Monk 1995; Lewis 1989; Lewis, Mayer 1987), higher grade level students (Van der Schoot et al. 2009), and lower-level grade students (Boonen, Jolles 2015; Mwangi, Sweller 1998; Schumacher, Fuchs 2012; Willis, Fuson 1988). These results raise a question as to whether younger students' difficulties with solving compare problems are caused by the formulations of IL problems or by a general lack of understanding of relations in both IL and CL problems.

Research that is somewhat more recent (Nesher, Hershovic, Novotna 2003) investigates compare problems with higher complexity. These problems include comparing three quantities instead of two and relations between them. As we previously stated, the difficulties with simple problems (with two entities) occur because of 1) language consistency, 2) lack of understanding of the symmetry of the operations, or 3) the different structure when the *referent* or the *compared* value is unknown. The difficulties are even greater on problems with higher complexity because there are two comparisons in a single problem. The results of research by Nesher et al. (2003) imply that students' achievement depends on the structure of each problem. It is not emphasized, but the examples used in this study integrated combine and compare problems, to which we refer as combine-compare problems. This integration enables the creation of numerous problems with the different structures. Similarly, when integrating combine and compare problems with two quantities (compared quantity and referent quantity), we can create problems with simple or complex structure.

METHODOLOGY

The study presented in this paper is a part of more extensive research that investigates the students' achievement on and strategies for combine-compare problems. The aim of the study is to investigate if the students' understanding of relational terminology (terms "more than" and "less than") develops with the students' age and if the development is accompanied by greater success in solving problems with more complex structures. For this purpose, we analyzed the subcategories of combine and compare word problems and made three integrations: 1) problem with CL formulation; 2) problem with IL formulation; 3) problem with complex structure. Specifically:

Problem A. CL formulation – CL compare problem and combine problem with an unknown combination (total) number:

Joca has 32 marbles, and David has 20 marbles more than him. How many do they have together?
Subproblem 1: Compare word problem with CL formulation
Finding the number of David's marbles
Subproblem 2: Combine word problem with an unknown combination
Finding the total number of marbles

Problem B. IL formulation – IL compare problem and combine problem with an unknown combination (total) number:

Zoka has 32 marbles, which is 20 marbles less than Angela. How many marbles do they have together?
Subproblem 1: Compare word problem with IL structure
Finding the number of Angela's marbles
Subproblem 2: Combine word problem with an unknown combination
Finding the total number of marbles

Problem C. Complex structure – compare problem and combine problem with an unknown subset:

Zoka and David have 84 marbles in total. David has 20 marbles more than Zoka. How many marbles does each child have?
Subproblem 1: Combine word problem with an unknown subset
Subproblem 2: Compare word problem

Problems A and B have a simple structure, and the difference between them is in the consistency of the language. Problem A is a CL problem that can be solved with the keyword method (Briars, Larkin 1984; Hegarty, Mayer, Monk 1995; Littlefield, Rieser 1993; Riley, Greeno, Heller 1983; Stigler, Lee, Stevenson 1990). Problem B is an IL problem whose solution implies knowing the symmetry of language and operations (Stern 1993). On the other side, Problem C has a more complex structure. It contains a combine problem with an unknown subset. The solution to this problem requires using more sophisticated strategies for solving. The language consistency is irrelevant in this integration.

The sample for our research consisted of 2nd, 4th, and 6th-grade students. Students at this age can use different problem-solving strategies: the 2nd graders are familiar with arithmetic strategies of solving; 4th graders can use basic algebraic notation and a small number of solving strategies, while 6th graders can use algebraic strategies for solving.

We operationalized the aim through the following research questions:

1. What is the achievement of students in certain grades (in 2nd, 4th and 6th grade) on CL and IL problems, and are there differences in the achievement on CL versus IL problems?

2. Is students' achievement on IL and CL problems related to the students' level of mathematics education (i.e., the grade students attend)?

3. Is students' achievement on the problem with the more complex structure related to the students' level of mathematics education (i.e., the grade students attend)?

4. What are students' strategies for solving combine-compare problems, and what are the most common mistakes they make?

Based on the results of previous research directed at students' achievement on combine and compare problems, we formulated the following hypotheses:

1. Students will have higher achievement on CL problems than on IL problems at all levels of education.

2. Students' achievement in solving IL and CL problems will be related to the level of students' education, especially on the problem with a more complex structure.

3. Students' achievement in solving the problem with a more complex structure will be related to the level of students' education, especially on the problem with a more complex structure.

4. Sixth-grade students will use algebraic strategies when solving problems with more complex structures, while younger students will use arithmetic strategies. We expect that the most common mistake will be using the keyword approach on IL problems and that younger students will use it more frequently.

The research sample consists of 134 students from one primary school in Belgrade that cooperates with the researchers' institution. Students are from two classes of 2nd grade (44 students), two classes of 4th grade (48 students), and two classes of 6th grade (42 students). They did not have a time limit to solve problems A, B, and C.

We used the Chi-square independence and homogeneity test to analyze the relationships between variables and differences in achievement. To express the strength of the association, we used Cramer's V coefficient.

RESULTS

The achievement of students on the CL problem (problem A) and IL problem (problem B) is presented in the Table 3.

Table 3. The students' achievement on CL and IL problems

Grade	Correct		Incorrect		Missing		Total
Consistency	CL	IL	CL	IL	CL	IL	
2	36	16	8	28	0	0	44
	81.8%	36.4%	18.2%	63.6%	0.0%	0.0%	100.0%
4	43	33	5	15	0	0	48
	89.6%	68.8%	10.4%	31.3%	0.0%	0.0%	100.0%
6	36	28	4	13	2	1	42
	85.7%	66.7%	9.5%	31.0%	4.8%	2.4%	100.0%
Total	115	77	17	56	2	1	134
	85.8%	57.5%	12.7%	41.8%	1.5%	0.7%	100.0%

We used the Chi-square homogeneity test to examine if there is a difference in the achievement on the CL versus the IL problem. The results of the test presented in Table 4 showed that students' achievement was significantly better on the CL than on the IL problem in 2nd grade (p = .000), in 4th grade (p = .012), and in 6th grade (p = .016).

Table 4. The results of the Chi-square homogeneity test in investigating the difference on CL versus IL problem

Grade	n	df	Chi square (n, df)	p
2	88	1	18.803	.000
4	96	1	6.316	.012
6	81	1	5.753	.016

Furthermore, we applied the Chi-square test of independence to examine the relationship between students' achievement on CL (and IL) problems and level of education. For the CL problem, the test did not show the existence of a significant relationship χ^2 (134, 2) = 1.658, p = 0.437. In Table 3, we can see that across the whole sample (134 students), about 85% of students solved CL problems correctly.

For the IL problem, the Chi-square test of independence showed a statistically significant relationship between the students' achievement and level of education χ^2 (133, 2) = 12.507, p = 0.002, with moderate strength of association r = .307 (Cramer's V coefficient). Further analysis of students' achievement (Table 5) showed that there is no significant difference in students' achievement in 4th and 6th grade (p = .963), but that there are differences between the students' achievement in 2nd grade versus 4th grade (p = .003) and 2nd grade versus 6th grade (p = .002). The results shown in the achievement table (Table 3) imply that about one-third of 2nd graders solved the IL problem correctly, and about two thirds of 4th graders and 6th graders solved this problem correctly.

Table 5. The Chi-square test results when examining the relationship between success and the age of students by grade pairs on a task with inconsistent wording (IL).

Comparison beetwen grades	Chi-square	p	r
2nd and 4th	χ^2 (92, 1) = 9.673	0.003	0.319
2nd and 6th	χ^2 (85, 1) = 8.665	0.002	0.324
4th and 6th	χ^2 (89, 1) = 0.002	0.963	

We have also analyzed the incorrect responses on CL and IL problems. The 2nd graders produced a greater number of errors on CL (18.2%, Table 3), and on IL (63.6%, Table 3) problems than 4th and 6th graders (who produced about 10% on CL and about 31% on IL problems, Table 3). In Table 6 we are presenting the analysis of incorrect responses on CL and IL problems. Students who did not solve the CL problem correctly mainly just added numbers from the text of the problem (11.4% of 2nd graders and 9.5% of 6th graders, while this number was smaller in 4th grade – 4.2%).

On the IL problem, 2nd graders also gave the biggest number of incorrect responses, but this time the error was in the relation term (50%, Table 6). The number of relation term errors was smaller in the 4th and 6th grades (31.3% and 26.2%).

Table 6. The categorization of students' incorrect responses on CL and IL problem

Grade	Uncategorized		Just add numbers		Relation term error		Total	
	CL	IL	CL	IL	CL	IL	CL	IL
2	3 6.8%	4 9.1%	5 11.4%	2 4.5%	/	22 50.0%	8 18.2%	28 63.6%
4	3 6.2%	0 0.0%	2 4.2%	0 0.0%	/	15 31.3%	5 10.4%	15 31.3%
6	0 0.0%	0 0.0%	4 9.5%	2 4.8%	/	11 26.2%	4 9.5%	13 31.0%

The students' achievement on the problem with complex structure is given in Table 7.

Table 7. Student achievement on a task with a more complex structure

Grade	Correct	Incorrect	Missing	Total
2	4 9.1%	34 77.3%	6 13.6%	44 100.0%
4	14 29.2%	21 43.8%	13 27.1%	48 100.0%
6	25 59.5%	15 35.7%	2 4.8%	42 100.0%
Total	21 15.7%	43 32.1%	70 52.2%	134 100.0%

The Chi-square test showed a significant relationship between students' achievement and age χ^2 (134.2) = 32.662, p = 0.000, with moderate strength of association r = .349 (Cramer's V coefficient). Further analysis showed that there are differences between every pair of different grades (Table 8). From Table 7 that shows students' achievements, it can be seen that almost 10% of 2nd graders, almost 30% of 4th graders, and almost 60% of 6th graders solved this problem correctly.

Table 8. The Chi-square test results when examining the relationship between success and age of students by grade pairs on a task with a complex structure

Comparison beetwen grades	Chi-square	p	r
2nd and 4th	χ^2 (92, 2) = 11.054	0.004	0.347
2nd and 6th	χ^2 (86, 2) = 24.541	0.000	0.534
4th and 6th	χ^2 (90, 2) = 11.822	0.003	0.362

Analysis of incorrect responses implies that students used superficial strategies. Second-grade students mainly used superficial strategies (almost 40%), while 4th and 6th-grade students made this mistake in roughly 8% and 7% of responses, respectively (Table 9).

Table 9. Students' mistakes when solving a task with a complex structure

Grade	Uncategorized	Superficial strategy	Error in relational term	Total
2	17 38.6%	17 38.6%	0 0.0%	34 77.3%
4	16 33.3%	4 8.3%	1 2.1%	21 43.8%
6	12 28.6%	3 7.1%	0 0.0%	15 35.7%

Both problems, with CL and IL formulation, could be solved using arithmetic strategy by performing arithmetical operations on numbers provided in the text of the problem. A task with a more complex structure allowed students to use different arithmetic and algebraic strategies. However, the algebraic strategy was used in a small number of cases: 2 (4.1%) 4th-grade students used an algebraic strategy, and 3 (7.1%) 6th-grade students (2 of whom only wrote the relations with symbols, then continued with the arithmetic strategy). However, we recognized different arithmetic strategies used by students:

1. Start from equal sets strategy, in which students start from the equal sets and make a difference between (e.g. 84 : 2 ± 10);
2. Start from the difference between sets strategy;
3. Guessing the quantities based on the solution of CL and IL problems.

The frequency and percentage of each strategy are provided in Table 10.

Table 10. Arithmetic strategies of students when solving a task with a complex structure

Grade	Start from equal sets	Start from the difference between sets	Guessing the quantities
2	1 2.3%	2 4.5%	0 0.0%
4	8 16.7%	7 14.6%	5 10.4%
6	11 26.2%	13 31.0%	7 16.7%

In addition to choosing a strategy, we were also interested in which strategies lead to the correct solution in most cases. Table 11 shows that the strategy that starts from the difference between the sets leads to the correct solution in more cases: 73% of the students who used this strategy solved the problem correctly, against 45% of the students who used the start from equal sets strategy.

Table 11. Choice of strategy and accuracy of completed tasks with a more complex structure on the entire sample (2nd, 4th and 6th-grade students)

	Start from equal sets	Start from the difference between sets
Correct	9 45.0%	16 72.7%
Incorrect	11 55.0%	6 27.3%
Total	20 100.0%	22 100.0%

DISCUSSION

Even though combine and compare problems and their integration are widespread in primary school mathematics, two aspects of integration need to be illuminated. First is the aspect of language consistency that compare problem brings to the integration, and the second is the problem of a more complex structure for cases in which the combine problem has an unknown subset.

Our first two research questions refer to language consistency – the achievement on CL and IL problems at different levels of education (2nd, 4th, and 6th grade) and the relationship between achievement and the levels. As we posed in the theoretical part of the paper, previous research in compare problem solving is mostly focused on the understanding of relational terms (Schumacher, Fuchs 2012; Riley, Greeno, Heller 1983; Riley, Greeno, 1988; Resnick 1983; Okamoto

1996; Okamoto, Case 1996; Cummins et al. 1988; Riley, Greeno, Heller 1983; Stern 1993) with emphasis on language consistency (Hegarty, Mayer, Green 1992; Hegarty, Mayer, Monk 1995; Stern 1993; Verschaffel 1994; Verschaffel, De Corte, Pauwels 1992; Lewis. Mayer 1987; Hegarty, Mayer, Green 1992; Lewis 1989; Verschaffel, De Corte, Pauwels 1992; Pape 2003; Van der Schoot et al. 2009). Our results showed that students have significantly higher achievement on the CL problem (Problem A) than on the IL problem (Problem B) in each level of education we investigated. This result is in accordance with previous research, which reports the consistency effect on different levels of education (Schumacher, Fuchs 2012; Riley, Greeno, Heller 1983; Riley, Greeno 1988; Resnick 1983; Okamoto 1996; Okamoto, Case 1996; Pape 2003; Van der Schoot et al. 2009; Hegarty, Mayer, Monk 1995; Lewis 1989; Lewis, Mayer 1987; Schumacher, Fuchs 2012; Willis, Fuson 1988).

Our results also showed no significant relationship between students' achievement on the CL problem and students' level of education. Regarding this result, it raises concern that about 15% of students in all grades did not solve the CL problem correctly (Table 3). These students do not have the conceptual knowledge required to solve this problem (Cummins et al. 1988; Riley, Greeno, Heller 1983). All the other problems that are more complex than the CL problem will stay out of their reach, which could cause difficulties in their future mathematics education.

The results of the analysis of the IL problem show significant differences between students' achievement and level of education. The consistency effect is the strongest in the 2nd grade (36% solved IL problem correctly, Table 3) but fades in the 4th grade (69% solved IL correctly, Table 3) and in the 6th grade (67% solved IL problem correctly, Table 3). Besides, the analysis showed no significant difference between 4th and 6th graders' achievement on the IL problem (Table 5). Surprisingly, two years of teaching algebra and arithmetic did not influence the level of understanding of relations between quantities.

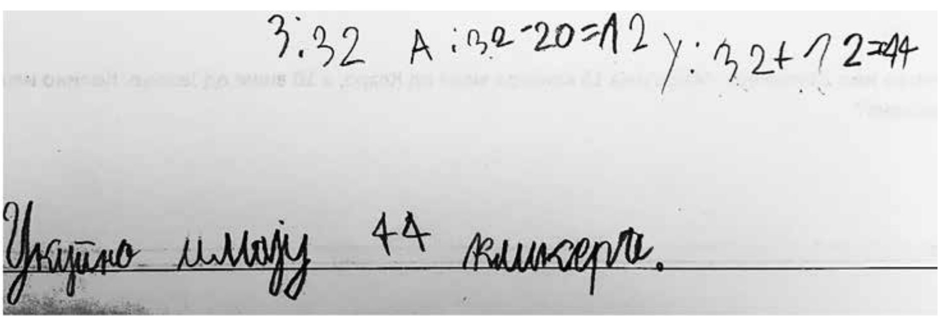
We found two possible guidelines in the literature for improving achievement. First, Boonen and Jolles (2015) showed that instruction focused on developing the relations' meaning and language symmetry can eliminate the consistency effect. Our results show that explicit instruction seems to be necessary at all levels of education, especially in the period from 4th to 6th grade, regardless of the results of the research that imply that the development of understanding continues (spontaneously) in adolescence (Wassenberg et al. 2008). Second, a body of research focuses on the benefits of graphical representations of problem structures by using diagrams (Willis, Fuson 1988; De Koning et al. 2022) and expressing relations in different ways (Stern 1993; Boonen, Jolles 2015; Schumacher Fuchs 2012; Riley et al. 1983; Riley, Greeno 1988; Resnick 1983; Okamoto 1996; Okamoto, Case 1996). These representations could be used to improve students' understanding and achievement and reduce the consistency effect at higher levels of education.

The analysis of students' incorrect responses also supports the conclusion that students did not develop a conceptual understanding of the relations needed for solving CL and IL compare problems (Hegarty, Mayer, Green 1992; Hegarty, Mayer, Monk 1995; Stern 1993; Verschaffel 1994; Verschaffel, De Corte, Pauwels 1992; Lewis, Mayer 1987; Lewis 1989; Pape 2003; Van der Schoot et al. 2009). Several students solved the CL problem by adding all the numbers in the text of the problem. Interestingly, this kind of incorrect response was seen more often in 2nd grade (11.4%, Table 6) and 6th grade (9.5%) than in 4th grade (4.2%).

This kind of reasoning is described in the literature as "compute first and think later" (Stigler, Lee, Stevenson 1990: 15), keyword method (Briars, Larkin 1984), or number grabbing (Littlefield, Rieser 1993). If the students read the term *more*, they would respond by adding two numbers, without considering the context of the situation. It is also interesting that students gave fewer incorrect responses of this kind on the IL problem (4.5%, 0%, 4,8%, respectively, in 2nd, 4th and 6th grade, Table 6).

As was expected, the greatest number of incorrect responses to the IL problem was rooted in the relational term. Half of the 2nd graders (50%, Table 6) made an error in the relational term, while slightly less than a third of 4th and 6th graders made this mistake (31% and 26.2%, respectively, Table 6). One of the examples is shown in Picture 1.

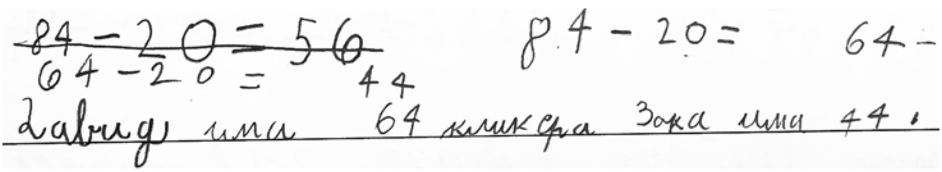
Picture 1. The incorrect relational term in students' responses



The combine-compare problem with a more complex structure (Problem C) is, in our opinion, cognitively challenging for 2nd graders. However, students in the 4th grade, especially in the 6th grade, should have a well-organized and flexible knowledge base that implies conceptual (e.g., using schematic representations for different types of problems) and procedural knowledge (formal and informal problem-solving strategies). Our results indeed show the statistical difference in the achievement of the 2nd, 4th, and 6th graders on this problem (Table 8), but, surprisingly, the success rate is low – slightly less than 10% of 2nd graders, slightly less than 30% of 4th graders, and slightly less than 60% of 6th graders solved the task correctly (Table 7).

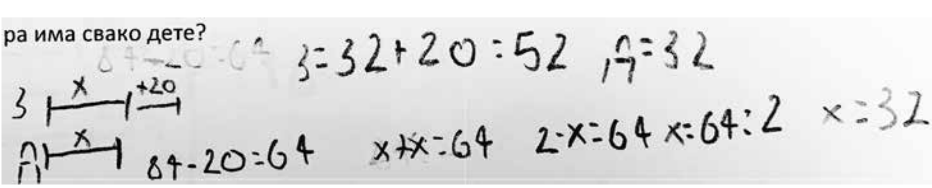
As expected, the incorrect responses show that the second graders had more difficulties with the problem with complex structure (77.3% of incorrect responses, Table 7) than students in 4th and 6th grade. They mostly tried to solve this problem using the keyword method (Briars, Larkin 1984) (38.6% of students, Table 9), as presented in the Picture. The 4th and the 6th graders mostly realized that the keyword method would not bring them to the correct solution; only 8.3% and 7.1% of students tried this method (Table 9).

Picture 2. Keyword method in solving problem with more complex structure

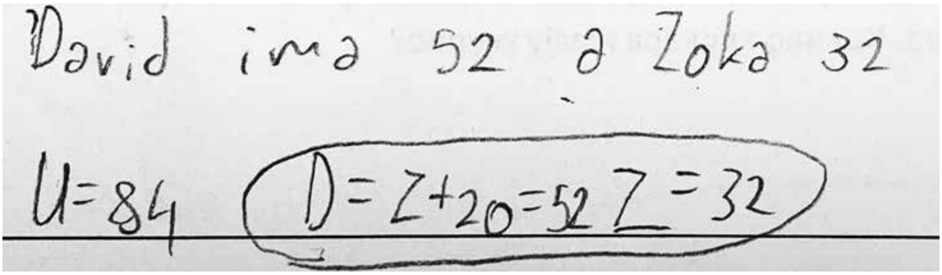


The analysis of students' strategies showed that only one student used graphical representation to solve the problem. This was not surprising for us because our previous research showed similar results (Zeljić, Dabić Boričić, Maričić 2021). On the other hand, it is surprising that only a few students used algebra to solve the problem – two of them in 4th grade and one in 6th grade (Picture 3). Two more 6th graders used algebraic symbols to represent relations in the problem, but they continued to solve it with arithmetic (Picture 4). Khng and Lee (2009) already noticed that many students return to arithmetic strategies of solving even if it was explicitly stated to solve the problem using equations. They consider using algebra for problem solving as moving forward to higher mathematics. Hence, students need to practice algebra even if they know how to solve the problem with the arithmetic method. We expected that 6th graders familiar with algebraic syntax and equation solving methods would use algebraic strategies for solving the problem with a more complex structure. In this context, the persistence in using arithmetic strategies could be considered an inhibition for further algebra learning.

Picture 3. Algebraic strategy and graphical representation in solving problem with more complex structure

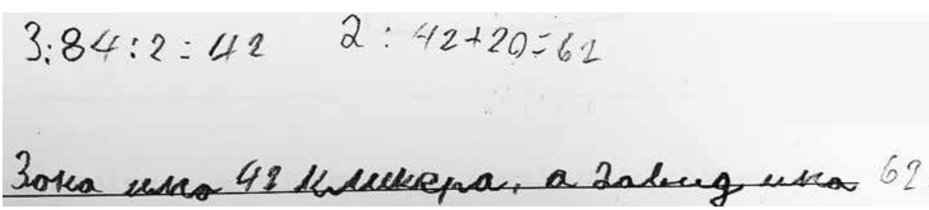


Picture 4. Recognized relations without algebraic strategy



For solving the problem with a more complex structure, students used two arithmetic strategies: 1) the one in which solving starts from the equal sets (computing 84: 2) and moves to the difference between them (by adding and subtracting 10); and 2) the strategy that starts from subtracting the difference and then making two equal sets. The second strategy was the strategy that led to the correct solution in greater numbers than the first one (Table 11). On the other side, some students who used the first strategy made one characteristic type of incorrect response. They started by making equal sets (dividing the total number of elements by 2), then added 20 to one set (Picture 5). They did not notice that the total number of elements does not fit the situation described in the problem. This solution shows that students do not have a coherent mental representation of all relevant elements and relations from the text of the problem (Hegarty, Mayer, Monk 1995; Pape 2003; Van der Schoot et al. 2009; De Koning et al. 2017; Koedinger, Nathan 2004) and that they do not apply modeling processes (Schwarzkopf 2007; Blum, Leiss 2007).

Picture 5. Incorrect the 'start from equal sets' strategy in solving problem with more complex structure



We can say that we confirmed the first hypothesis: students are more successful in solving problems with consistent language formulation. The second hypothesis is disproved: 1) There was no significant relationship between students' achievement on the CL problem and students' level of education; 2) We did not find differences in achievement of 4th and 6th graders on the IL problem (4th graders solved IL and CL better than 6th graders). The third hypothesis was related to solving the problem with a more complex structure, and it is confirmed: there was a significant difference in the achievement between 2nd, 4th, and 6th-grade students. Contrary to expectations, 6th graders did not use algebraic strategies in solving the problem with a more complex structure (4th hypothesis), and there are no differences in students' choice of strategy depending on their level of education. The most frequent mistake was the mistake in the understanding of relational terms, and it was based on keyword strategy.

CONCLUSION

In this research, we investigated achievement on and strategies for solving problems in which relational terms and language consistency are important. We looked into the possible effect of age/grade on achievement on three problems: the CL problem, the IL problem, and the problem with complex structure, which integrates simpler compare and combine word problems. Our results are in accordance with previous research, namely that students' achievement is better on the CL problem (with no difference between grades) than on the IL problem. There is a statistically significant difference in achievement between 2nd grade and 4th and 6th grade. However, there is no difference between 4th and 6th grade, which implies that there is a need for instructional intervention regarding understanding relational terms and problem-solving strategies. The most common strategy that led to an incorrect solution was the superficial strategy of using the keyword method. Students showed low achievement on the problem with more complex structure, though there is a significant difference between grades, with 6th graders being the best. Two solving strategies for this task stand out, one being the 'start from equal sets' strategy and the other 'start from the difference between sets' strategy, out of which the second strategy led to correct solution in more cases. Surprisingly, even though this problem is suitable for algebraic solving strategy or using diagrams, very few students used algebraic strategy, and only one student used graphical representation to solve the problem.

Understanding the problem-solving process is a very complex issue. Awareness of the different aspects of understanding and solving text problems can help us identify students' obstacles when trying to solve them. Instructions for understanding compare problems, which are based on verbal instructions and the use of diagrams and schemes, are still being developed and have not been implemented in educational practice. Several researchers have argued that the stereotypical nature of word problems in traditional textbooks encourages students to use superficial solving strategies, such as the keyword approach, without building an adequate model of the situation described in the problem. Students need rich experience with different semantic structures of tasks. The nature and structure of problems affect how students reason and can limit or expand understanding of mathematical concepts. Only the systematic use of all types of tasks and the planning of the solving process as an application of mathematical modeling leads to the ability of students to solve different types of mathematical tasks.

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РАЗУМЕВАЊЕ РЕЛАЦИОНИХ ТЕРМИНА У ПРОБЛЕМИМА ПОРЕЂЕЊА-КОМБИНАВАЊА НА РАЗЛИЧИТИМ НИВОИМА ОБРАЗОВАЊА

Резиме: Једно од централних истраживачких питања у математичком образовању последњих деценија било је питање употребе текстуалних проблема са реалистичним контекстом, пошто у настави математике имају широку примену. У овом раду бавимо се *рутинским проблемима* који могу доприносити развоју концептуалног знања о основним рачунским операцијама. Определили смо се за испитивање успешности ученика на проблемима који настају интеграцијом проблема комбиновања и поређења. Иако су обе врсте проблема темељно истражене у литератури, нисмо наишли на истраживања која се баве успешношћу ученика на интегрисаним

проблемима, иако су овакви проблеми присутни у уџбеницима и наставној пракси. Њихов значај је, осим што доприносе концептуалном разумевању операција, у томе што могу указати на ниво развијености релационе терминологије код ученика, стратегије решавања проблема, као и на спремност за употребу графичких репрезентација. Интегрисани проблеми комбиновања и поређења могу бити различитих језичких формулација и нивоа комплексности. Проблеми поређења у овој интеграцији могу бити конзистентне и неконзистентне језичке формулације, док проблеми комбиновања могу допринети усложњавању структуре проблема. Стога смо истраживали успешност ученика на три различита типа проблема: 1) проблем са конзистентном језичком формулацијом, 2) проблем са неконзистентном језичком формулацијом, 3) проблем са сложенијом структуром. Претходна истраживања, која су се бавила истраживањем појединачних типова проблема, показала су да ученици имају највише потешкоћа у решавању проблема поређења, као и да су успешнији у решавању проблема поређења са конзистентном у односу на неконзистентну језичку формулацију. Резултати представљени у овом раду су део већег истраживања чији је циљ да испита да ли се релациона терминологија (термини „за толико више" и „за толико мање") развија са нивоом математичког образовања ученика и да утврди да ли овај развој прати и већи успех у решавању проблема са комплекснијом структуром. Стога наш узорак чине ученици другог, четвртог и шестог разреда. Истраживачка питања на која одговарамо у овом раду односе се на разлике у постигнућима ученика (другог, четвртог и шестог разреда) на интегрисаним проблемима са конзистентном и са неконзистентном језичком формулацијом, на везу између успешности ученика у решавању ових проблема и њиховог узраста (нивоа математичког образовања), на везу између успешности ученика у решавању задатка са комплекснијом структуром и њиховог узраста, као и на стратегије и честе грешке при решавању ових проблема. Узорак у истраживању су чинили ученици школа у Београду, и то 44 ученика другог, 48 ученика четвртог и 42 ученика шестог разреда. Резултати су потврдили резултате претходних истраживања – да су ученици успешнији у решавању проблема са конзистентном него са неконзистентном језичком формулацијом. Интересантан је резултат да нема разлике у успешности у решавању задатка са конзистентном језичком формулацијом између ученика другог, четвртог и шестог разреда – на целом узорку просечна успешност у решавању овог задатка је око 85%. То значи да око 15% ученика на свим истраживаним нивоима образовања имају потешкоће са разумевањем релационе терминологије у њеној најједноставнијој језичкој формулацији. Резултати су такође показали да на проблему са неконзистентном језичком формулацијом не постоје разлике у успешности између ученика четвртог и шестог разреда, што може упућивати на застој у развоју разумевања релационе терминологије након увођења алгебре у математичко образовање, а самим тим и на потребу за више инструкционих интервенција у на овом узрасту. Резултати ученика на задатку са сложенијом структуром показали су да постоје разлике у успешности ученика на различитим нивоима образовања. Очекивано, најмање успешни су били ученици другог разреда, затим четвртог, док су најуспешнији били ученици шестог разреда. Очекивано, ученици другог и четвртог разреда нису користили алгебарске стратегије решавања, а изненађујуће је да ни ученици шестог разреда нису користили алгебарске стратегије. Овај резултат потврђује мишљење многих аутора да треба инсистирати и на алгебарским стратегијама решавања проблема иако ученици умеју да га реше

аритметичком стратегијом. Анализом одговора ученика на овај задатак препознато је да је „метод кључне речи” најчешће водио ученике ка нетачном решењу. Такође су препознате и две стратегије решавања проблема – она која „полази од једнаких скупова” и она која „полази од разлике међу скуповима”, при чему је друга стратегија у већем броју случајева водила према тачном решењу. Такође, ученици нису корстили сликовне репрезентације у решавању овог проблема, иако је проблем био погодан за њихово коришћење. Истраживачи су раније приметили да стереотипско коришћење текстуалних проблема у традиционалним уџбеницима подстиче ученике да користе површинске стратегије решавања, као што је метод кључне речи. Стога је потребно обогатити искуства ученика са проблемима различите семантичке структуре, чиме се утиче на процес њиховог мишљења и разумевања математичких концепата. Систематском употребом свих врста задатака и стратегија које примењују процес математичког моделовања може се утицати на побољшање постигнућа ученика у решавању свих типова математичких проблема.

Кључне речи: текстуални проблеми, проблеми комбиновања, проблеми поређења, стратегије решавања проблема, математичко образовање.