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PEDAGOGICAL PERSPECTIVE ONILL-DEFINED MATHEMATICAL PROBLEMS

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Abstract: *Traditionally teaching mathematics in schools is based primarily on well-posed problems. Intuitively, there is a clear difference between them and ill-defined problems. Research in the field of problem-solving is often based on the analysis of procedures for solving incompletely posed problems (i.e. ill-defined problems) because they are expected to provide a deeper insight into mathematical abilities as well as the ability of critical and creative thinking of respondents. However, ill-defined problems are scarcely explored as a main subject of interest. This paper aims to investigate the place of those types of problems in teaching mathematics. As a context of the investigation, we consider various classifications of math problems. Three classifications of mathematical problems with a "pedagogical perspective" are elected to be analyzed using the theoretical epistemological method of comparative analysis. All three classifications consider the position of participants in problem-solving, teacher and students. As a result of the analysis, we create a comparison of the spectrums of problems, particularly paying attention to ill-defined problems in them. The discussion finishes addressing the place and purpose of using ill-defined problems in mathematics instructions.*

Keywords: *task classification, problem posing, problem space, (in)completely posed problems*

Introduction

Problem-solving is at the center of learning mathematics in elementary school. Mathematical tasks are the basic content of teaching mathematics in primary and secondary school through which mathematical knowledge is adopted, exercised, systematized, applied, assessed, and evaluated. There are numerous studies on problem-solving, while a significantly smaller number of studies is devoted to researching problems themselves (Bonotto, 2007;

Polya, 1973; Stanic & Kilpatrick,1988; Shoenfeld,1992). Knowledge of problem-solving but also of problem-posing skills acquired in mathematics classes is applied in other domains in and out of school through mathematical modeling. For example, Pollack (Pollack, 1988) illustrates the importance of mathematics for engineers stating that engineers need to know beyond oral and written techniques of calculus and critical reasoning, they need to be able to perform mathematical modeling which starts with problem posing. Kilpatrick (Kilpatrick, 1987) recognizes problem-posing and problem-solving skills as objectives of mathematics teaching. Because of the significance of those abilities, it is important to consider the range of problems and the place of ill-defined problems.

The topic of "posing mathematical problems" began out of studies focused on other research areas such as problem-solving, mathematical competencies of students, assessment of mathematical knowledge and mathematical teaching, and methodological training of prospective math teachers (Lavy & Shriki, 2007; Kruteskii,1976; Silver, Mamon-Downs, Leung Kenney, 1996; Crespo, 2003; Nesher,1980; Reusser,1988). For example, the project Cognitively Guided Instructions (CGI) systematically researched how children solve problems and as a result, they came up with a classification of problem types (Carpenter, 1998). Some authors have pointed out how important is the choice of problems for the development of the ability to reason critically (Bandjur, 1999; Maričić, Špijunović & Lazić, 2015). Maričić and colleagues found that in their sample of third-grade students, a selection of tasks helped them to develop the general ability of critical reasoning but also supported the development of various skills associated with problem-posing: formulation of the problem, problem reformulation, problem evaluation, and sensitivity to the type of problem (Maričić et al., 2015). Silver and colleagues (Silver et al., 1996) investigated how teachers pose problems by identifying classes of generated problems. On a class of open problems, in which the limiting conditions in the problem space changed, researchers registered that some teachers kept the terms of the problem and changed questions, while others varied the problems by changing what is known in the problem.

Contemporary school mathematics, which is within the constructivist theory of mathematics education encourages posing open problems and investigations which provide rich opportunities to build conceptual understanding and development of procedural fluency (von Glasersfeld, 1995). In realistic mathematics education problematization (identifying problems within explored situations) is the basis of gaining new knowledge. The new Serbian curriculum for primary and secondary education points to the need for attending to the problems in a realistic context.

Problem space

Let's attend to the epistemological meaning of basic concepts related to the analysis of problem space. Problems solving in mathematics science may be described as an investigation that starts from the given conditions and facts in an attempt to prove the truthfulness of a statement. Problem-solving is somewhat different in school. Schoenfeld cites Vebster's dictionary according to which "In (school) mathematics, (it is) any requirement to be met or requirement to do something," and "The question is confusing or difficult" (Shoenfeld, 1992, p. 337). In mathematics teaching, mathematical tasks are requirements (usually in a form of questions) that are resolved using mathematical tools (procedures, procedures, logical-mathematical reasoning). Along the line, Foster uses the term mathematical task for "any question or requirement that provokes mathematical thought " (Foster, 2015, p. 4).

Which of the math problems can be called problems? Hendersons and Pingry (Hendersons & Pingry, 1953) list the following conditions for the task to be identified as problem 1. The solver is aware of the clearly set goal and whose achievement is desired; 2. There is an obstacle on the way to obtaining the goal, and the known algorithm or usual procedures are insufficient for the removal of obstacles to obtain the goal; 3. The problem solver's deeper thinking can help him understand the problem more or less clearly, identify various possibilities (alternative routes) and test their feasibility. The problem is not considered in isolation from the solver. The second condition, that the problems are associated with cognitive effort is commonly identified (Yeo, 2007, Milinkovic, 2015, Reys, Lindquist, Lambdin, Smith and Suydam, 2001, Schoenfeld, 1985). Rays notes the need for creative effort and higher cognitive functions in problem-solving, while Schoenfeld points out that effort is primarily intellectual (and not "technical", as is the effort to apply the skill of performing computational procedures) in the solution process. Milinković underlines the limited possibility of an objective assessment of whether a task is a problem or not. She distinguishes mathematical problems from other tasks based on the level of cognitive demand, pointing out that so-called routine arithmetic tasks can be problems if the method of solving is unknown. A problem on one level of schooling can become a routine task at the next level. Thus, the identification of problems is related to the prior knowledge, experience, and abilities of problem solvers at the time of problem-solving (Milinković, 2015). This relativistic view Yeo (2007) calls a pedagogical perspective, as the identification of the problem (versus simple task) is based on the perspective of the student and his inability to directly apply known procedures to arrive at a solution.

The analysis of mathematical tasks implies consideration of conditions (known and unknown, i.e. required quantities), their relations and goals, as well as the theoretical basis, the procedure for solving, and discussion of the solution. The goal of solving effort is most often to determine unknown quantities, connections, and properties, but it can also be to draw conclusions and justify claims or to display quantities and relationships in construction tasks in geometry. The theoretical basis for problem-solving is mathematical knowledge needed to determine the relationship between quantities and arriving at a procedure (algorithm) for coming to a solution. Thus the process of problem-solving is a sequence of steps from known quantities to finding answers to questions. The solution of a task ends with a discussion that includes interpretation and verification (Dejić & Egerić, 2003; Špijunović and Maričić, 2016).

Each problem is set in the space of the problem, with its formulation and structure. Milinković (Milinković, 2015) indicates that every problem can be described via its context, the known and unknown elements of the entities (quantities), and the relationship between the elements. In psychology, problem space is defined as a mental representation of a problem that contains knowledge about the initial and target state (solution) of the problem, as well as all possible intermediate states that must be passed in order to establish a connection between the beginning and target state (solution).

The context of the problem determines the boundaries of the problem's space. The context can be abstract and real. The problem consists of the context (conditions) and relations between the known and unknown values. Constructs presented in the analysis problem are the structure of the problem, method of the problem, and solution. The structure of the problem consists of its formulation and representation, i.e. the way in which it is given and the way in which it is presented. The formulation specifies the elements of the problem space and their relationships in a given context. The problem can be set at different levels of abstraction in the form of pictures, diagrams, or text. The task format underlines the order of transfer from external to internal representations (Silver, et al., 2011). Presenting the problem situation with different representations encourages flexibility in the choice of representations in students. As a result, they become more able to deal with the problems involving mathematical modeling (Friedlander and Tabach, 2001). Finally, the solution of the problem is an answer to the question and it is not necessarily single.

Well-defined problems and ill-defined problems

Although the terms ill-defined (or incomplete set problems) and completely set (or fully posed problems, or well-defined problems, or well-structured problems) are used in the methodical textbooks (Dejić and Egerić, 2003; Špijunović and Maricic, 2016) in Serbia and Springer Encyclopedia of Mathematical Education (Lerman, 2014), or monographs and survey papers dealing with problem-solving and problem-posing (Singer, et al., 2015; Grouws, 1992) the meaning of these terms is not specified. Rather, the meanings of these contrasting terms are implied, intuitively established. The term well-posed problem appears in the Encyclopedia of Mathematical Education in the description of research on modern teaching of mathematics aimed at solving problems, including ". . . the focus is on presenting and practicing well familiar method for solving well- placed to problems . . . . ”(Lerman, 2014, p. 644). In the case of a well-posed problem, it is known what is required and a question is posed. In others, some elements are missing such as a) the question is not explicitly stated, but is expected to be intuitively recognized by the solver based on the analysis of the problem space, or b) some elements of the problem space are missing and different possibilities must be considered and in accordance with these alternatives. Some of the ill-defined problems could be incorrectly formulated (e.g. when a given relation between the values is impossible in a given context). Discussion about how well the problem is designed involves analysis of the problem’s space and the possibility for finding the answer with one or more correct answers. For example, in the case of a problem involving solving inequality, only the correct answer is the one that includes all solutions. Becker and Shimada (1997) described a type of task in which there were multiple correct solutions, although that did not mean that there were multiple correct answers. For example, solving a quadratic equation can produce two correct solutions, but that is the only correct answer, because if only one solution is given when there are two solutions, then the answer is wrong. Incompletely defined problems are sometimes terminologically identified with open problems, a term that primarily refers to problems that are not explicitly clear by which procedure or method they are solved.

Methodology

The subject of research is the classifications of mathematical problems.

The theoretical epistemological method of comparative analysis was applied (Miljević, 2007). The process of collecting and analyzing scientific resources displayed a variety of general classifications of mathematical problems and critical perspectives on the place and function of ill-defined problems in mathematical instructions. Research studies in the recent past deal with the analysis of classes of mathematical problems from the aspect of problem design (Maker and Schiever, 1991; Maker, 1993; 2001; Milinković, 2015; Yeo, 2007). Yeo’s, Foster’s, and Maker-Schiever's problem classifications were selected (Yeo, 2007; Foster, 2015; Maker and Schiever, 1991). The comparison of the three identified classifications in our research is conducted with three objectives:

1. pedagogical analysis of the position of subjects (teacher, students) in the problem-solving process,

2. analysis of the place of ill-defined problems in the context of the teaching process,

3. establishing the purpose of ill-defined problems in contemporary mathematical education.

Results

We have created the following examples of mathematical problems we will rely upon in the analysis:
Example 1 Calculate the circumference of an isosceles triangle whose base is a=5cm and b=7cm.
Example 2 The sum of three consecutive numbers is 3726. Which method would you use to
determine those numbers?
Example 3 Determine the number of required columns for the fence of a rectangular field whose
dimensions are a=50m, b=70m.
Example 4 Construct an equilateral triangle ABC whose side a = 5cm, and then triangle AVO
symmetric to the triangle ABC with respect to side AV.
Example 5 Determine the next member in the sequence 1, 2, 4, 8, ___.
Example 6 Make a model of a rocket whose parts will be in the shape of a rectangle, a square, and a triangle.
Example 7 Determine the optimal price of a school snack.
Example 8 Investigate the numbers that contain the number 6.
Example 9 The pizzeria sells Mini, Medium, and Large pizzas. Mini pizza has 6 slices
and costs 540 dinars, Medium pizza has 8 pieces and costs 640 dinars, and Large pizza has 10 pieces and costs 770 dinars. a) If you wanted to buy the cheapest piece, which pizza slice would you choose from? b) What questions can you answer based on the data?
Example 10. "How can you graphically display the results of a survey "Favorite cartoon"
Problem 11 Make a model of a rocket from a model of geometric bodies.

Classification by Yeo

The starting point for the classification of Yeo (Yeo, 2007) is in our t - national purpose on the basis that the tasks can be divided into two broad classes: 1) mathematically "rich" tasks and 2) tasks that are not mathematically rich. The group of mathematically rich tasks includes analytical and synthesis tasks that can provide opportunities for gaining new knowledge and developing mathematical knowledge related to procedural and mathematical technical knowledge such as problem-solving strategies, analytical thinking, metacognition, and creativity. Characteristic for the later class are procedural tasks that are useful for practicing mathematical procedures. Yeo points out that often the classification of problems from school textbooks is based on the methods of solving those tasks. In the first group are "routine tasks" which include a wide range of tasks that can be called "standard textbook assignments" or "procedural problems" which are used routinely for practicing procedural skills students need to acquire (Example 1). Such tasks may be problematic for the students who do not know the procedure (e.g. missing certain steps) or do not know the standard procedure at all. The second class includes mathematical problems that aim to apply a particular method of solving problems (Example 2). In the third class are "research problems" that do not have a clear goal (Example 3). A problem solver needs to set the goal and the process of problem-solving is open and complex by nature. A problem that may be qualified as belonging to “research problems” may become a second class task if a teacher takes an active role in problem-solving. The problem is reduced to the application of a certain method of solving problems by teachers' involvement. Yet, it can also be deepened into a real “research problem" by expanding the scope of the problem. As a special group of mathematical tasks, Yeo singles out problem-posing tasks because he believes that this type of task enables students to manifest knowledge and creativity. In the last class are “project problems” which can include research tasks with or without mathematical context as well as realistic problems (Example 7).

Classification by Foster

Foster proposes a classification of problems based on the fact that different students or even the same students do not experience one task in exactly the same way at different times (Foster, 2015). This observation is the basis of the classification problems metaphorically described as "different rays" which are refracted through the concave, convex, or concave-convex lenses which can be convergent, divergent, convergent-divergent. In this case, like in Yeo’s classification, identification of a problem occurs not only on the basis of the formulation of the problem but also on the role of a teacher in the problem-solving process. In a class of the convergent problems are problems that have a correct answer which can be reached by a variety of methods (Example 6). In a such task, there is a unique tendency in the process of solving, and all students' reasoning is directed towards a similar approach to solving problems. Divergent problems are open and pupils use various approaches in the process of problem-solving (Example 7). Metaphorically, as well as light rays diverge, from vertical starting positions, the position of "ray of light" represent the diversity of the starting points of different students wherein the " more extreme " problem requirements bent to a greater extent than those nearer the center (similarly the path of rays), while “less extreme” requirements deviate very little from their “natural” path (Foster, 2015). In the case of convergent-divergent problems, the question that belongs to one or the other type of task changes alternatively (Example 9). Foster emphasizes the role of the teacher who can lead the discussion in different directions. In addition, he notes that individually, students can choose different ways of solving problems.

Classification by Makers and Schiever

Maker and Schiever established six types of school problems, ordered hierarchically. The classification is based on an assessment of how much are teachers and students familiar with a problem (Maker and Schiever 1991; Bahar Maker, 2015). This classification was developed as part of the “Discoveries” project (Maker, 1993, 2001; Maker and Schiever, 1991). They state that the structure of each mathematical problem corresponds to one of six types of problems, in the scale range from "type I" problem to “type VI”. In the “type, I” problem both teacher and students recognize the problem from the past and know how to solve it and the teacher knows at least one correct solution. Examples of this type of problem are the most common in teaching because this type of problem includes solving mathematical problems by known procedures, using a formula, algorithm, or well-known method (Example 1) . In Type II" problems students recognize that class of problems, but do not know the way of solving I while the teacher knows both methods of solving the problem and the solution. Problems belonging to “type II " are structurally close to "type I" problems except that the studentc do not know the way they could come up with a solution. Examples of such type of problem are mathematical “story problems” that require from solver to understand and apply an appropriate problem-solving method (Example 3). In the next level, "type III" problems, both the teacher and students recognize the class of a problem which can be solved with multiple alternative methods known to the teacher. A typical example of this type of problem is the task of constructing a figure with a given property (Example 4). “Type IV" problem has for students and the teacher recognizable structure and multiple known procedures for solving it. The problem has multiple correct answers known to the teacher. Often such problems are solved by induction and have a range of correct answers. Geometric problems which can be solved by manipulation or task of writing different equations using three given numbers and arithmetic operations are examples of "type IV " problems. One such example of a task is determining the rule for patterns' growth based on the given members of the array or the task of constructing a figure with a given property (Example 5). "Type V" problems structure is known to the teacher and students but neither the teacher nor students know a method for solving it. These problems are completely structured, but the methods and solutions are open and the problem can have infinitely many solutions or none at all. Typical examples of this type of problem are examples 6, 10, and 11. In them, all parameters are given, yet there is no unique solution for either of them. Finally, problems "type VI" are those which are not fully structured(defined), given in a form which is for the teacher and students unknown and therefore the method of solving and solution (or solutions) are also an unknown. A typical problem of this type is the most complex; in order to find a method for solving it, often it is necessary to reformulate the problem or to find a different representation from the initial one. Such problems, as a rule, has several possible solutions. These are often realistic problems arising from more or less complex life situations (Realistic Mathematics Education problems that can be described in multiple ways in the process of mathematical modeling. Examples of "type VI" problem situations are "taxi problem" (Determining an optimal price list of taxi services) or "eco problem" (Creating a mathematical model for solving pollution within the local community) (Example 7).

If we compare the types of problems presented in the previous section, we can see that the " type I " problem is completely structured and closed, while the " type VI " problem is incompletely structured. All mathematical problems fall somewhere between those two ending points on a problem structure scale. In this typology, the intention of the author was to find a place for each task on the " continuum of tasks" from " type I " to " type a VI " , although it is possible to simplify or further complicate the division. Problem " Type I " can be solved in only one way in a particular context, a solver is not allowed to know the right method or procedure to arrive at the correct solution (egg. to know the formula) . The " type I " problem has one correct answer. On the other hand, the " type VI " problem is not known to either the teacher or students and there could be an infinite number of ways to reach a solution. The solver must determine which method(s) may be better than the others and whether any of the methods is appropriate in the given context, where there is always possible that there is no appropriate method because there is no single solution to the problem. The " type V " problem is so abstract that it may or may not have an infinite number of possible solutions. Solution of problem "type VI" is often subjective, prone to different interpretations based on other factors (e.g. political).

Discussion

In the previous sections, we presented three different classifications of mathematical problems curatively presented in Table 1. Initially, the starting point of all three classifications was problem space. All three classifications are grounded in the pedagogical perspective because the types of problems were determined and described based on the position of participants in the problem-solving. In each of these classifications, the tasks are considered in relation to the mathematical contents, structure, and formulation.

Substantial differences between Yeo's classification on one side and Maker-Schiever's and Foster's on the other side is that the latter classifications are relativistic and the Yeo's is not. Thus, the context of instruction, primarily the pedagogical and mathematical knowledge of the participants in problem-solving activity effectively change relative problem's difficulty. Pedagogical support, primarily didactical interference in problem-solving, directs the process and changing the problem solving process and students 'perception of the problem. For Foster, the social component is important, but it is even more important to notice the diversity of students' positions with regard to their mathematical knowledge and metacognitive characteristics. In contrast, Yeo is focused on a problem structure and mathematical content within it . On the other hand, Maker and Schiever as well as Foster emphasize social component, the importance of the participants (knowledge and position) rather than problem formulation, structure or math content of the problem as criteria for classification. Although Yeo's classification is not relativistic he also points to critical impact of teacher in the pedagogical guidance of students that can change the nature of the problem (actually, problem solver's perception) in the problem solving process.

All three classifications describe problems that are incompletely formulated i.e. ill-defined (Table 1). It is noticeable that the problems in all classifications, those types of problems are considered to be of higher-order on the scale. For example, in Yeo's classification, procedural tasks, considered of lower math value ("scarce" value) is completely defined while on the opposite side are "synthetic tasks" that arise in the process of critical analysis of a situation and are defined along the line of the modeling process. Obviously, in all three classifications, incompletely set problems are those that require higher-level cognitive processes, creative thinking, and deeper knowledge of mathematical content.

Completely formulated problems represent the basic corpus of math tasks that students encounter during regular math classes. Ill/defined problems are in contrast, sporadically present in various forms, particularly in out-of-school math programs, preparations for competitions, etc.

Table 1 Problem continuum matrix

Typology	Type	Assignment method		Method		The solution		Problem setting
1.(Yeo)	Tasks that are not mathematically rich (Not provocative, which are not mathematically attractive)	Procedural (routine)		Application of known procedures, techniques Application of known methods Development of new strategies				Fully formulated
	Mathematical " rich " tasks (challenging, layered)	Analytic		Explorative Projects (mathematical project, realistic project)				(Not) fully formulated
		Synthetically		Explorative				Incompletely formulated
				Problem posing				Incompletely formulated
2. (Foster)	convergent	Closed		Familiar, unique Familiar, various		Known and unique		Fully formulated (structured)
	divergent	Open		Various		Various		Incompletely defined
	convergent divergent	combined		Various		Unique/ Various		(Not) completely Defined
3 . (Maker-Schiever)		Teacher (T)	Student (S)	T	S	T	S	
	I	+	+	+	+	+	-	Fully formulated
	II	+	+	+	-	+	-	Fully formulated
	III	+	+	+ / -	-	+	-	Fully formulated
	IV	+	+	+/–	-	+/–	-	Fully formulated
	V	+	+	-	-	-	-	Fully formulated
	VI	-	-	-	-	-	-	Not fully formulated

The didactical shift toward Realistic Mathematics Education in the Serbian curriculum produced theoretical support for changing teaching practice and incorporating projects and research problems in the dominantly procedural corpus of tasks. Time is also in regular classes obtained space for incomplete set s problems. This should undoubtedly be reflected in the textbook literature as well. The fact is, however, that this is not the case. The basis for this assumption is the fact that even today the standards for the quality of a mathematical textbook are such that incompletely formulated problems are considered wrong or incomplete. (Note, however, that not all task formulations are acceptable, i.e. that there are indeed incorrectly formulated tasks or tasks with oversight, without the necessary data, or with contradictory data and therefore unsolvable.)

The presented classifications differ in the place and function assigned to well-defined and ill-defined problems. On one side, completely posed problems are dominantly recognized as useful in learning and practicing techniques, procedures, and methods. On the other, ill-defined problems have a function in the development of general and specific strategies, new methods, metacognition, creativity, and critical thinking. Due to the different functions of these two types of problems, well-defined problems dominate in school mathematics instructions. Realistic mathematics education, promote changes and a significant presence of ill-defined problems in school practice.

Finally, some research questions can be identified :

1. How well are teachers prepared for dealing with such ill-defined problems?

2. How much are incompletely posed problems are present in the classroom practice and professional literature?

3. What are the effects of introducing ill-defined problems at different stages of school?

Conclusion

We based our argument on the foundation of the theoretical contributions in domains of problem-posing and problem-solving. We presented, illustrated, analyzed, and discussed similarities and differences among three classifications of mathematical problems proposed by Yea, Foster, and Maker-Schiever. These classifications had a pedagogical perspective that emphasizes the significance of knowledge of participants in the problem-solving process - teacher and students. All classifications dealt with ill-defined problems and placed them in a higher place on a scale as they require and promote critical thinking, the flexibility of reasoning, and a creative approach to problem posing. It was concluded that didactical orientation toward realistic mathematics education gives theoretical support for more variability in the selection of mathematical problems in school curriculum which primarily uses resources with rich collections of well-defined procedural problems rather than research and design problems. The effects of using ill-defined problems in school practice need to be verified in future research. The didactical shift toward Realistic Mathematics Education in the Serbian curriculum produced theoretical support for changing teaching practice and incorporating projects and research problems in the dominantly procedural corpus of tasks. Time is also in regular classes obtained space for incomplete set s problems. This should undoubtedly be reflected in the textbook literature as well. The fact is, however, that this is not the case. The basis for this assumption is the fact that even today the standards for the quality of a mathematical textbook are such that incompletely formulated problems are considered wrong or incomplete. (Note, however, that not all task formulations are acceptable, i.e. that there are indeed incorrectly formulated tasks or tasks with oversight, without the necessary data, or with contradictory data and therefore unsolvable.)

The presented classifications differ in the place and function assigned to well-defined and ill-defined problems. On one side, completely posed problems are dominantly recognized as useful in learning and practicing techniques, procedures, and methods. On the other, ill-defined problems have a function in the development of general and specific strategies, new methods, metacognition, creativity, and critical thinking. Due to the different functions of these two types of problems, well-defined problems dominate in school mathematics instructions. Realistic mathematics education, promote changes and a significant presence of ill-defined problems in school practice.

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