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How do precious and industrial metals hedge oil in a multi-frequency semiparametric CVaR portfolio?

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ABSTRACT

This paper tries to reduce extreme risk of Brent oil by constructing multivariate portfolios with precious and industrial metals in a multi-frequency framework. Extreme risk is measured by the parametric CVaR and more elaborate semiparametric CVaR measure, while the wavelet technique is used to build portfolios in different time-horizons. The results indicate that gold dominates in the precious metal portfolios, creating the lowest downside risk in most cases. The back-testing results reveal that the raw data CVaR portfolio with precious metals is the best, while the midterm mCVaR portfolio with industrial metals has upper hand. In terms of forecasting, CVaR portfolios with precious and industrial metals are the best in the short- and midterm horizons, respectively. In the long-term horizon, none of the portfolios is good in back-testing and forecasting, which means that CVaR and mCVaR models are not adequate risk functions for identifying realized risk in the long term. In the oil-dominated portfolios, the CVaR portfolios with gold are the best in terms of the lowest risk as well as back-testing and forecasting performances. When Brent is replaced by WTI oil, the share of WTI is somewhat smaller in the portfolios because WTI has higher risk. When the US 10Y bond is added to the portfolios, the portfolio risk is reduced because the bonds are very weakly correlated with the commodities.

1. Introduction

Oil, as the most important commodity in the world, is very susceptible to various factors, such as global economic crisis, regional wars, the strengthening of the dollar and the unstable demand and supply of oil (see e.g. Obadi, 2010; Bein & Aga, 2016; Estrada et al., 2020; Ozcelebi, 2021). The recent events of the COVID pandemic and the Ukrainian war have significantly affected the oil market, causing tremendous oil price oscillations in a relatively short period of time. In particular, at the begging of the pandemic, oil price plummeted from 68.9 USD per barrel in January 2020 to 19.9 USD in April 2020, while after the Russian invasion of Ukraine, the price of a barrel exceeded 123 USD per barrel in June 2022 (see left plot in Fig. 1). These developments generate heavy losses for agents who work with oil (see right plot in Fig. 1), which is why the topic of oil hedging is widely discussed among oil investors and portfolio managers. In order to respond to adverse oil price changes, investors are looking for alternative asset combinations with oil, where such portfolios could mitigate extreme risk of oil, and tradable metals play an important role in this matter (Adekoya & Oliyide, 2020). This is because metals have two favourable characteristics – they are readily available in major markets including London, New York, Tokyo, etc., while, on the other hand, metals behave intrinsically different relative to oil, which induces low correlation between the

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assets (Cai et al., 2020).

According to the above, this paper addresses the topic of oil hedging from several different aspects, some of which are being presented for the first time. In particular, the paper constructs two portfolios of five assets, combining Brent oil futures with four precious metals futures (gold, silver, platinum and palladium) and four industrial metals futures (aluminium, copper, nickel and zinc). Futures are preferred over spot prices for several reasons. First, futures markets process new information more quickly, making asset prices more realistic. Daily futures prices are very similar to daily spot prices, hence the results of the constructed portfolios can be easily applied to portfolios with spot prices, which is useful for oil producers or oil traders. Also, futures are suitable for portfolio diversification because futures do not require the purchase of physical assets, but only contracts that could be closed easily. Combining futures in a portfolio is similar to portfolios with stocks, which means that portfolio theory can be applied to futures market with no problem, according to You and Daigler (2013). Referring to Adekoya and Oliyide (2020), empirical studies have largely focused on precious metals, while relatively few papers researched hedging potential of industrial metals. They also cast doubt on whether precious metals are good hedging instruments for oil, which opens a room for comparing hedging capabilities of both precious and industrial metals. Based on our knowledge, only Živkov et al. (2022) researched oil hedging with precious metals in a multivariate portfolio, but they did not consider industrial metals, and they focused on minimum variance, while we try to minimize extreme risk of oil. Due to the fact that existing literature was mainly focused on precious metals (see e.g. Hernandez et al., 2019; Huang et al., 2022; Das et al., 2022; Ahmed et al., 2022), and much less on industrial metals, the first contribution of this study is to fill the missing gap regarding the comparative ability of precious and industrial metals to hedge oil risk in a multivariate portfolio.

The second aspect of this research refers to risk evaluation in a multi-asset portfolio, where a new methodological approach is proposed, addressing the well-known problem of risk measurement bias. In other words, most papers considered risk-evaluation from the viewpoint of common variance. However, variance gives equal weight to positive and negative returns, which could distort the level of risk because investors are interested only in negative returns, which is called downside risk. In order to overcome this measurement bias, parametric Value-at-Risk (VaR) was introduced by JP Morgan in 1994. Parametric VaR overcomes the problem of positive returns by observing only a specific quantile at the left tail of the standard normal distribution (Chen et al., 2015). In other words, VaR calculates the risk of loss, where this risk is more extreme when probability is higher. This paper looks at the very edge of the left tail of the portfolio, meaning that the downside risk is calculated at 99 % probability level, which is referred as extreme risk. However, VaR is not an ideal risk measure because it has some undesirable theoretical properties such as the lack of subadditivity and non-convexity, which can create multiple local optima and unstable VaR rankings (Li et al., 2012). Even more important is inability of VaR to measure the losses beyond the threshold amount of VaR (Snoussi & El-Aroui, 2012), and this could create misleading investment decisions. In order to address this issue, Rockafellar and Uryasev (2002) proposed parametric conditional VaR (CVaR), which controls the magnitude of losses beyond VaR. In other words, if 99 % probability is assumed, than CVaR calculates the average loss in the worst 1 % of returns (see Živkov et al., 2021).

However, both parametric VaR and CVaR are valid only if the distribution follows the Gaussian function, which is a very strict and unlikely assumption that can lead to erroneous risk assessment if not met. Daily commodity time-series do not usually follow a normal distribution, and this could produce a misleading measure of CVaR. This happens because CVaR uses only the first two moments to estimate extreme losses, while the third and fourth moments are neglected. In order to find a solution that overcomes the two-moment bias, we refer to Favre and Galeano (2002), who addressed this issue by introducing semiparametric or modified VaR (mVaR). This risk function is based on the Cornish-Fisher expansion (Cornish & Fisher, 1938), where the third and fourth moments play a role in calculating downside risk. Therefore, in order to accurately calculate the level of extreme loss, we merge a very complex semiparametric CVaR algorithm with the multivariate portfolio optimization procedure. Semiparametric CVaR penalizes unfavourable characteristics of the distribution, such as negative skewness and high kurtosis, and rewords positive features, such as positive skewness and low kurtosis, and this is where the superiority of semiparametric risk measures comes to the fore (see e.g. Chai & Zhou, 2018).

The third aspect of the paper deals with another important topic, which is the preference of investors to different investment time-horizon. In other words, market participants implement various investment strategies depending on their goals, where the length of the investment horizon plays a significant role. However, investigation of economic phenomena in different time-horizons is generally little studied due to the sample reduction problem that emerges when researchers try to observe different time-horizons in daily data. In this regard, the construction of the portfolios that pursue different time-horizons is even scarcer, and we are aware of only three papers in this type of research – Mensi et al. (2019), Mensi et al. (2021a) and Alqaralleh and Canepa (2022). However, all these studies designed two-asset portfolios, while we make five-asset portfolios, which is a more complicated task. In order to make portfolios in

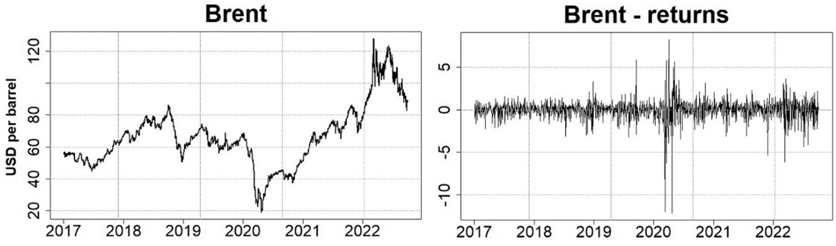

Fig. 1. Empirical dynamics of Brent oil.

different time-horizons, we refer to the aforementioned papers and use the Maximum Overlap Discrete Wavelet Transformation (MODWT) methodology to create wavelet signals representing medium and long-term horizons. For the short time-horizon, raw data is used. The wavelet method is convenient because it preserves information stored in the empirical data as Cai et al. (2021) stated.

The novelty of this paper involves combining the wavelet approach with the portfolio optimization procedure that has never been done before, according to our best knowledge, and this is where we find the motive for this research. Merging these two different methodologies allow us to inspect whether and how portfolio structure varies when different time-horizons are in focus. This question is not unreasonable, as the size of downside risk metric differs when calculated at various wavelet scales (see Mensi et al., 2021b). This happens because the wavelet signals at higher scales are smoother and without outliers, which implies lower CVaR and mCVaR values, and this could affect the structure of the constructed portfolios. In other words, the goal is to empirically assess which risk model (CVaR or mCVaR) better estimates the realized extreme downside risk at different frequency scales. If the portfolios have high negative skewness and high kurtosis, there is no doubt that the CVaR will be significantly lower than the mCVaR counterpart, the only question is how much. To be more formal in determining which risk function is more accurate in recognizing empirical risk levels, we employ the standard coverage test of Kupiec (1995) for back-testing and forecasting evaluations.

In the oil hedging process, we construct optimal portfolios, but also oil-dominated portfolios, with the 50 % oil constraint. This is done for practical reasons as there is a good chance that oil could be excluded from a multi-asset portfolio because oil is the riskiest asset, and these results mean nothing to agents who work with large quantities of oil on a daily basis. Therefore, imposing constraint on portfolios could indicate which metals in the portfolio with oil provide the best hedging results over different time-horizons. Two complementary analyses are also performed. The first checks how robust the results are if WTI oil is part of the portfolio instead of Brent. The second tries to answer how he structure of the portfolio and its performance changes if long-term bonds are added to the portfolio.

Besides introduction, the rest of the paper is structured as follows. Second section gives literature overview. Third section is reserved for the used methodologies – MODWT methodology and portfolio optimization process. Fourth section presents dataset and preliminary findings. Fifth section shows the results of the estimated optimal and oil-dominated portfolios. Sixth section is reserved for the two complementary analyses, while the last section concludes.

2. Literature review

Hedging oil with metals is not new in the international literature, but much more attention is given to precious metals than industrial metals. For instance, Hernandez et al. (2019) researched how precious metals (gold and silver) and agricultural commodities (wheat, corn, and rice) can be used to diversify and hedge investment in WTI oil under extreme downside and upside oil market scenarios. They found that extreme lower quantiles of oil returns have positive effect on the lower quantiles of gold, silver, and rice returns. This means that decrease in oil returns during a bearish oil market will cause a decrease in precious metal and rice returns, which means that these commodities cannot be used to hedge the downside risk of oil investments. This is especially true in short term. Yıldırım et al. (2022) examined risk transmission between oil and precious metal markets induced by the COVID-19 pandemic using the DCC-GARCH model. They found significant risk transmission between oil prices and precious metal prices, particularly during the onset of the COVID-19 pandemic. They reported that investors and portfolio managers should consider gold and silver because these two offer the best diversification and hedging opportunities. Huang et al. (2022) investigated whether the green bond acts as a hedge or safe haven to crude oil in extreme market conditions, also comparing the results with precious metals. They found green bond as strong safe haven and hedge for the crude oil market, while gold is weak safe haven and strong hedge. Silver only acts as weak hedge, whereas other precious metals are neither safe haven nor hedge. The paper of Shafiullah et al. (2021) researched long-run dependence and causality between oil and precious metals across quantiles. They found that causality running from oil to metal prices is quantile-dependent and differs according to the metal, whereas upward and downward movements in metal prices have no causal effect on oil prices. Based on the results, they asserted that oil and gold/silver markets are disconnected at times of downward market movements, meaning that investors can use those commodities to protect themselves against risk arising from extreme downward price movements.

As for industrial metals, very few papers researched the hedging capabilities of these assets in combination with oil. For instance, Adekoya and Oliyide (2020) analysed the effectiveness of seven commonly traded industrial metals in providing cover for investors against oil market risks. For oil market risks, they used four different oil shocks (oil inventory demand shocks, oil supply shocks, oil consumption demand shocks and economic activity oil shocks). They found that the nature of shocks, whether demand- or supply-based, determines the hedging ability of the industrial metals. The asserted that all three demand-based oil shocks can be effectively hedged by virtually all the metals, while only the oil supply shocks cannot be hedged by the metals. The paper of Chen et al. (2022) examined how asymmetric oil price shocks affect non-ferrous metal market under different market conditions. The results show that the nexus of non-ferrous metal prices with oil price shocks and uncertainty are distinct under different market conditions, where the effects caused by oil price shocks are stronger in a bear market than in a bull market. They contended that investors can couple non-ferrous metals and oil into the investment portfolios to reduce investment risks especially during the crisis. Mensi et al. (2021c) researched the dependence structure, risk spillovers and conditional diversification benefits between oil and six non-ferrous metals futures markets (aluminium, copper, lead, nickel, tin, and zinc), using a variety of copula functions and Conditional Value at Risk (CoVaR) measure. They found significant lower tail dependence and upper tail independence between oil and non-ferrous metals markets. Based on the results, they reported that aluminium and nickel have similar conditional diversification benefits, while lead, tin and zinc are also closed. The conditional diversification benefits is higher for nickel regardless of the portfolio composition and the probability level. They asserted that diversification gains decrease during stress market periods.

3. Research methodologies

3.1. MODWT approach

This paper tries to find out how the structure of portfolios looks like if investors pursue different investment horizons. Also, in this way we can measure the level of downside risks when different time-horizons are in focus. For this purpose, we use MODWT technique to decompose empirical time-series in the several wavelet time–frequency components. Wavelet methodology can provide an appropriate trade-off between resolution in the time and frequency domains, which is different from the traditional Fourier analysis that only emphasizes frequency domain. Two basic wavelet functions are in the core of wavelet theory – the father wavelet (ϕ) and the mother wavelet (ψ). Father wavelets augment the representation of the smooth or low frequency parts of a signal with an integral equal to 1, while the mother wavelets describe the details of high frequency components with an integral equal to 0 (Compains et al., 2021). In particular, father wavelet describes the long-term trend over the scale of time-series, while the mother wavelet defines fluctuations around the trend. Two functions of father $\phi_{J,k}(t)$ and mother $\psi_{j,k}(t)$ wavelet are generated in the following way:

$$\phi_{J,k}(t) = 2^{-J/2}\phi\left(\frac{t-2^J k}{2^J}\right), \psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \tag{1}$$

Symbol j refers to wavelet level, the scale or dilation factor is 2^j, whereas the translation or location parameter is $2^j k$. As much as j grows, so does the scale factor 2^j, which is a measure of the width of the functions $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$, and it affects the underlying functions to get shorter and more dilated. Besides, when j gets larger, the translation steps automatically increase in order to accommodate the level of scale parameter 2^j.

As for this research, the MODWT algorithm is used, which characterises a highly redundant non-orthogonal transformation. In this regard, the signal-decomposing procedure in MODWT[1] is presented in the following way:

$$s_{J,k} \approx \int f(\mathrm{t})\phi_{J,k}(t)dt \tag{2}$$

$$d_{j,k} \approx \int f(\mathrm{t})\psi_{j,k}(t)dt, j = 1, 2, \cdots, J \tag{3}$$

where symbols $s_{J,k}$ and $d_{j,k}$ stand for the fluctuation and scaling coefficients, respectively, at the j^{th} wavelet level that reconstructs the signal in terms of a specific frequency (trending and fluctuation components). Accordingly, an empirical time series y(t) can be expressed in terms of those signals as:

$$\mathrm{y(t)} = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \ldots + \sum_k d_{1,k}\psi_{1,k}(t), \tag{4}$$

3.2. Portfolio optimization with CVaR and mCVaR minimization objectives

After wavelet deconstruction, we combine Brent oil with four precious and industrial metals in a multivariate portfolio, where goals are minimum CVaR and mCVaR risk measures. All portfolios are constructed in a multi-frequency setting, emphasizing short-, mid- and long-term horizons. In particular, we combine the two complex risk algorithms with the portfolio optimization procedure of Markowitz (1952). According to our best knowledge, the existing papers have only been used parametric CVaR (see e.g. Vo et al., 2019; Braiek et al., 2020; Shen et al., 2021; Luan et al., 2022), while complex portfolio construction where all four moments are taken into account has never been attempted thus far. Making both CVaR and mCVaR portfolios, allow us to compare the level of risk between the two portfolios and determine whether and how much CVaR underestimates downside risk.

First step in explaining how the minimum CVaR and mCVaR portfolios are created is showing the process of minimum-variance portfolio optimization:

$$min\sigma_p^2 = min\sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j} \tag{5}$$

where σ_p^2 is portfolio variance, σ_i^2 is variance of a particular asset i, w_i denotes calculated weight of an asset i in a portfolio. $\rho_{i,j}$ is a correlation coefficient between the particular pair of assets (i and j).

Every multivariate portfolio must comply with the fact that the sum of all asset-weights in a portfolio is equal to one, while all asset-weights are somewhere in between zero and one.

$$\sum_{i=1}^{N} w_i = 1; 0 \le w_i \le 1 \tag{6}$$

[1] MODWT transformation was done via'waveslim' package in'R' software.

Also, every portfolio with minimum variance has the corresponding mean value, which is weighted average portfolio return (r_p), and it can be calculated as in Eq. (7).

$$r_p = \sum_{i=1}^{n} w_i r_i \tag{7}$$

For parametric VaR calculation, the first and second moments (r_p and σ_p) from Eqs. (7) and (5), respectively, are necessary elements: $VaR_p = r_p + Z_\alpha \sigma_p$. Z_α is the left quantile of the normal standard distribution. Finding the integral of VaR is the way of calculating CVaR, which is like in Eq. (8).

$$CVaR_\alpha = -\frac{1}{\alpha}\int_0^\alpha VaR(x)dx \tag{8}$$

Expression (9) shows how the minimum CVaR portfolio can be optimized.

$$\min CVaR_p(w), \sum_{i=1}^{n} w_i r_i \tag{9}$$

However, the level of downside risk, calculated via parametric CVaR can be biased because it takes into account only the first two moments. The problem can be solved by considering all four moments of a distribution, and this is where mCVaR comes to the fore. Analogous to CVaR, mCVaR is the integral of mVaR, while mVaR is calculated as: $mVaR_\alpha = r_p + Z_{CF,\alpha}\sigma_p$, where $Z_{CF,\alpha}$ is the non-normal-distribution percentile adjusted for the higher-order moment information, according to the Cornish–Fisher Expansion in Eq. (10):

$$Z_{CF,\alpha} = Z_\alpha + \frac{1}{6}(Z_\alpha^2 - 1)S + \frac{1}{24}(Z_\alpha^3 - 3Z_\alpha)K - \frac{1}{36}(2Z_\alpha^3 - 5Z_\alpha)S^2 \tag{10}$$

S and K stands for measures of skewness and kurtosis of a portfolio. Similar to expression (9), the minimum semiparametric CVaR portfolio can be optimized as in expression (11):

$$\min mCVaR_p(w), \sum_{i=1}^{n} w_i r_i \tag{11}$$

In order to quantitatively estimate how much the downside risks of Brent oil are reduced in the metal portfolios, we calculate Hedge Effectiveness Indices (HEI). Therefore, taking into account specific risk measure (RM), i.e. CVaR or mCVaR, HEI_{RM} can be calculated in the following way:

$$HEI_{RM} = \frac{RM_{oil} - RM_{portfolio}}{RM_{oil}} \tag{12}$$

4. Dataset and preliminary findings

The study uses daily near maturity futures from the Chicago mercantile exchange of Brent oil and the selected metals – precious metals (gold, silver, platinum and palladium) and industrial metals (aluminium, copper, nickel and zinc). Futures involve buying and selling contracts, not physical assets, which makes futures more suitable for hedging purposes. We separately combine Brent oil with each group of metals, creating in this way the two five-asset portfolios. Data-span covers the period between January 2017 and October 2022, and all the futures commodities are collected from the investing.com website. All the time-series are transformed into log-returns (r_i) according to the expression: $r_i = 100 \times log(P_{i,t}/P_{i,t-1})$, where P_i is the price of a particular asset. This study is unique because it applies complex multivariate portfolio optimization in different time-horizons - short, mid and long-term. To this end, empirical daily data represent the short-term horizon, while MODWT technique is used to create wavelet details denoting the midterm and long-term horizons. In particular, we use D5 for the midterm horizon and D7 for the long-term horizon, where D5 scale denotes the time-horizon between 32 and 64 days, while D7 scale indicates the time-horizon between 128 and 256 days. In order to preserve space, we show only daily returns of gold as well as transformed D5 and D7 wavelet returns in Fig. 2.

Table 1 presents descriptive statistics of the selected commodities, containing the first four moments of the raw data and the two

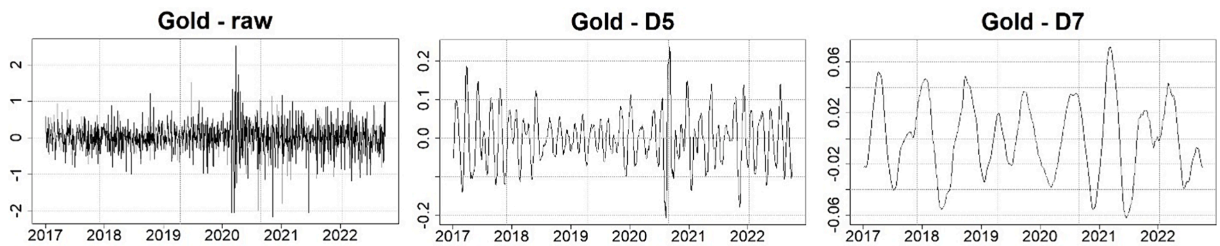

Fig. 2. Raw returns and D5 and D7 wavelet returns of gold.

wavelet scales. In this way, the four-moment properties of the time-series are scrutinized, which can be used later on in explaining structures of the constructed portfolios. In particular, the first noticeable difference between raw data and the wavelet transformed time-series is the fact that standard deviation gets lower as wavelet scale increases, which is in line with the paper of Mensi et al. (2021b). This happens because deviations from the mean decreases as wavelet scales get larger, which is obvious in Fig. 2. This characteristic can affect the CVaR portfolios because the second moment is the key factor determining the level of CVaR. In other words, it can be expected that portfolios in longer time-horizons have much smaller CVaR risk. Of all the assets, gold has by far the lowest second moment, which is particularly true for raw data and D5 wavelet scale. This means that gold could have the highest share in both CVaR and mCVaR portfolios.

On the other hand, mCVaR takes into account all four moments and the difference between the three time-horizons is not as straightforward as in the case of the second moment. In other words, negative skewness at higher wavelet scales turns to positive, as in three of the four cases of the precious metals at D7 wavelet scale and all cases of the industrial metals at D5 wavelet scale. Positive skewness is good characteristic for investors because it reduces mCVaR risk, and the assets with these characteristics could have higher weight in the mCVaR portfolio. On the other hand, kurtosis is lower at higher wavelet scales, which signals the absence of extreme values. In much of the cases, kurtosis at D5 and D7 wavelet scales turns to negative values, and this indicates platykurtic distribution, which has flatter peak and thinner tails compared to a normal distribution. This is also a positive characteristic for investors, and this could favour assets with this feature in the mCVaR portfolio.

Besides four-moment properties of the time-series, mutual correlation between the assets also plays an important role in the portfolio optimization process. Therefore, Table 2 calculates Pearson correlation between the assets, taking into account raw data and the two wavelet scales. According to Table 2, Brent has relatively weak correlation with the metals at the raw data level, which means that both groups of metals are potentially good hedging instruments for Brent. It is interesting to note that pairwise correlations in industrial metals portfolio increase as wavelet scales get larger. This happens because idiosyncratic properties of the time-series disappear at higher wavelet scales, making wavelet correlations stronger (see e.g. Abid & Kaffel, 2018). In the case of precious metals, pairwise correlations decrease at D7 scale compared to D5 scale, indicating that the time-series get more diverged at higher wavelet scale. This phenomenon also can be found in the existing literature (see e.g. Tiwari et al., 2013). Therefore, combination of the results in Tables 1 and 2 should provide a reasonable explanation for the structure of the created portfolios.

5. Empirical results

5.1. Optimal portfolio construction

This section presents the results of the constructed optimal multi-frequency portfolios, where Panels A and C in Table 3 contains calculated shares of the assets in both CVaR and mCVaR portfolios, while Panels B and D show portfolio risk performances. For each multi-frequency portfolio, a rational explanation is given regarding which factors determine a specific structure of the portfolio.

In the short-term horizon, both CVaR and mCVaR raw data portfolios with precious metals have the same structure, where gold takes a very high share of 92 %, while the rest goes to Brent oil. Gold has the highest share in the CVaR portfolio due to the lowest second moment, while the situation is repeated in the mCVaR portfolio because gold also reports the lowest third and fourth moments. Brent oil has 8 % in both portfolios, although it has very high standard deviation, skewness and kurtosis, but Brent has the lowest pairwise correlation with gold (0.112), which explains its share in the portfolios. All other precious metals underperform gold in terms of second, third and fourth moments as well as pairwise correlations, making gold the dominant asset in the portfolio, while all other metals have a zero shares. We only know the paper of Živkov et al. (2022), who combined precious metal with oil in a multi-asset portfolio, and they also found a very high proportion of gold in combination with Brent oil, which is in line very much with our results.

On the other hand, in the short-term CVaR portfolio with industrial metals, the highest share have aluminium and copper with 43 %

Table 1
Descriptive statistics of the selected assets in different frequency scales.

	Raw data				D5 wavelet scale				D7 wavelet scale			
	Mean	St.dev.	Skew.	Kurt.	Mean	St.dev.	Skew.	Kurt.	Mean	St.dev.	Skew.	Kurt.
Brent	0.014	1.156	−1.497	21.082	0.000	0.188	−0.583	9.072	0.000	0.132	−0.190	3.454
Precious metals												
Gold	0.011	0.392	−0.270	5.473	0.000	0.066	0.145	0.321	0.000	0.030	0.027	−0.765
Silver	0.008	0.784	−0.584	7.720	0.000	0.143	−0.006	1.707	0.000	0.075	0.257	0.597
Platinum	−0.004	0.749	−0.424	6.136	0.000	0.123	−0.241	2.162	0.000	0.052	0.007	−0.417
Palladium	0.028	1.010	−0.672	12.366	0.000	0.166	−0.214	5.882	0.000	0.085	−0.089	0.449
Industrial metals												
Aluminium	0.010	0.556	−0.209	2.642	0.000	0.089	0.048	1.104	0.000	0.057	−0.110	2.268
Copper	0.010	0.548	−0.395	2.036	0.000	0.107	0.117	1.034	0.000	0.053	−0.315	−0.016
Nickel	0.023	1.357	2.811	155.5	0.000	0.183	0.509	6.839	0.000	0.100	0.028	0.849
Zinc	0.005	0.660	−0.156	1.385	0.000	0.133	0.145	1.698	0.000	0.056	−0.279	−0.467

Table 2
Estimated Pearson correlations in different frequency scales.

Raw data

	Brent	Gold	Silver	Platin.	Palladi.		Brent	Alumi.	Copper	Nickel	Zinc
Brent	1	0.112	0.180	0.226	0.237	Brent	1	0.219	0.304	0.209	0.231
Gold	0.112	1	0.782	0.536	0.345	Alumi.	0.219	1	0.482	0.186	0.420
Silver	0.180	0.782	1	0.599	0.401	Copper	0.304	0.482	1	0.305	0.588
Platin.	0.226	0.536	0.599	1	0.517	Nickel	0.209	0.186	0.305	1	0.345
Palladi.	0.237	0.345	0.401	0.517	1	Zinc	0.231	0.420	0.588	0.345	1

D5 wavelet scale

	Brent	Gold	Silver	Platin.	Palladi.		Brent	Alumi.	Copper	Nickel	Zinc
Brent	1	0.312	0.537	0.523	0.512	Brent	1	0.315	0.340	0.328	0.194
Gold	0.312	1	0.813	0.572	0.496	Alumi.	0.315	1	0.609	0.387	0.548
Silver	0.537	0.813	1	0.750	0.577	Copper	0.340	0.609	1	0.464	0.691
Platin.	0.523	0.572	0.750	1	0.644	Nickel	0.328	0.387	0.464	1	0.355
Palladi.	0.512	0.496	0.577	0.644	1	Zinc	0.194	0.548	0.691	0.355	1

D7 wavelet scale

	Brent	Gold	Silver	Platin.	Palladi.		Brent	Alumi.	Copper	Nickel	Zinc
Brent	1	−0.078	0.498	0.615	0.109	Brent	1	0.524	0.723	0.215	0.561
Gold	−0.078	1	0.671	0.138	0.437	Alumi.	0.524	1	0.724	0.650	0.713
Silver	0.498	0.671	1	0.548	0.485	Copper	0.723	0.724	1	0.491	0.865
Platin.	0.615	0.138	0.548	1	0.198	Nickel	0.215	0.650	0.491	1	0.571
Palladi.	0.109	0.437	0.485	0.198	1	Zinc	0.561	0.713	0.865	0.571	1

Table 3
Optimal portfolio structures in different frequency scales.

	Raw data		D5		D7	
	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR
Panel A: Portfolios with precious metals						
Brent oil	8 %	8 %	2 %	0 %	0 %	0 %
Gold	92 %	92 %	98 %	100 %	79 %	72 %
Silver	0 %	0 %	0 %	0 %	0 %	0 %
Platinum	0 %	0 %	0 %	0 %	21 %	28 %
Palladium	0 %	0 %	0 %	0 %	0 %	0 %
Panel B: Performance of portfolios with precious metals						
Risk measure	−1.018	−2.051	−0.177	−0.176	−0.073	−0.059
HEI	0.671	0.863	0.646	0.874	0.793	0.901
Panel C: Portfolios with industrial metals						
Brent oil	5 %	0 %	5 %	0 %	0 %	0 %
Aluminium	43 %	39 %	70 %	58 %	32 %	0 %
Copper	38 %	42 %	25 %	31 %	50 %	27 %
Nickel	0 %	0 %	0 %	11 %	0 %	0 %
Zinc	14 %	19 %	0 %	0 %	18 %	73 %
Panel D: Performance of portfolios with industrial metals						
Risk measure	−1.242	−1.639	−0.227	−0.220	−0.134	−0.147
HEI	0.598	0.893	0.543	0.832	0.625	0.734

Note: Greyed values indicate higher HEI, comparing the precious and industrial metals portfolios.

and 38 %, respectively, whereas in the mCVaR portfolio, they switch places, and now copper has the highest share with 42 %, while aluminium follows with 39 %. These two metals have the lowest second moment, according to Table 1, which is the key reason for their high share in the CVaR portfolio. Low standard deviation probably plays a crucial role in the mCVaR portfolio, because aluminium and copper have significantly higher skewness and kurtosis than zinc, but, in spite of that, they have twice the proportion of zinc in the mCVaR portfolio. Brent has relatively low share of 5 % only in the CVaR portfolio probably because it has relatively low correlation with aluminium (0.2019). In the mCVaR portfolio, the share of Brent reduces to zero, while zinc increases to 19 % due to the lowest third and fourth moments.

In the midterm horizon, the structure of the portfolios significantly changes, which justifies the analysis of the multi-frequency portfolios. In the precious metals portfolio, gold increases its share to 98 % and 100 % where target functions are CVaR and mCVaR, respectively. Gold has by far the lowest second moment in the D5 scale, and also the lowest correlation with Brent oil (0.312), which are the reasons why gold has very high share in the CVaR portfolio. On the other hand, gold has positive skewness (0.145) and the lowest kurtosis (0.321) at D5 scale, which makes gold the only asset in the mCVaR portfolio.

In the industrial metals portfolio, aluminium increases its share to 70 % in the CVaR portfolio, because it has the lowest second moment (0.089), while copper follows with 25 % due to the second lowest standard deviation (0.107). Brent has 5 % owing to relatively low pairwise correlations with aluminium and copper, the two dominant assets in the CVaR portfolio. In the mCVaR portfolio, aluminium reduces to 58 %, while copper increases to 31 % because aluminium has higher kurtosis (1.104) than copper (1.034). Nickel reports 11 % probably because of the highest positive skewness (0.509) and relatively low pairwise correlations with aluminium (0.387) and copper (0.464). Brent has no share in the D5 mCVaR portfolio because it has the highest negative skewness (−0.583) and the highest kurtosis (9.072).

Changes in the portfolio structures are also evident in the long-time horizon, which is represented by the D7 scale. In other words, gold and platinum are the only instruments in the CVaR precious metals portfolio, with 79 % and 21 % share, respectively. This is because gold and platinum have the lowest second moment in the long time-horizon, 0.030 and 0.052, respectively. These two also have negative kurtosis, which lowers mCVaR risk, and puts these two metals at the first two places in the mCVaR portfolio, with 72 % and 28 %.

As for the long-term industrial metals portfolio, copper and aluminium have the two highest shares in the CVaR portfolios with 50 % and 32 % because they have standard deviations of 0.053 and 0.057, respectively. Zink has even lower second moment than aluminium in amount of 0.056, but zinc has higher pairwise correlation with the most dominant copper (0.865), than aluminium has with copper (0.713), which puts zinc to the third place with 18 %. However, in the mCVaR portfolio, situation changes significantly, in a sense that zinc emerges at the first place with 73 %, while copper is the second one with 27 %. The reason is relatively high negative kurtosis of zinc (−0.467) and lower negative skewness (−0.219) compared to negative skewness of copper (−0.315). Brent has no share in the long term portfolios with precious and industrial metals.

Panels B and D reveal performances of the constructed portfolios in terms of risk measures and HEI values. These results show dominance of the precious metals portfolios in the five out of six cases, while only the mCVaR portfolio with industrial metals has upper hand in the short time-horizon. Gold is the primary reason why the precious metals portfolio performs so well. Some papers, which used precious and industrial metals for hedging are in line with our findings. For instance, AlKhazali et al. (2021) used the stochastic dominance approach, trying to determine whether the gold-oil portfolio stochastically dominates the oil portfolio. They asserted that the gold-oil portfolio is better than the one without gold, where portfolio risk decreases as more gold is added into the oil portfolios. The paper of Wang et al. (2022) researched the hedging ability of gold against oil risk in different time horizons using wavelet methodology. They contended that gold has very good hedging results versus extreme oil prices in the medium and long-term horizons, which concur very well with our results. On the other hand, industrial metals report better hedging results in the short-term mCVaR portfolio, which concurs with the assertion of Adekoya and Oliyide (2020), who argued that industrial metals can hedge successfully oil shocks.

Comparing the results based only on the risk performance is only one aspect of the analysis, and this does not guarantee superior hedging strategy because this tells nothing about statistical inference (Su et al., 2023). In this regard, we carry out the in-sample back-testing and out-of-sample forecasting analysis of the portfolios, and the results are presented in Table 4. Referring to Chai and Zhou (2018), we use the standard coverage test of Kupiec (1995) to validate whether the models are adequate and accurate. From the aspect of back-testing, this procedure checks whether the realized losses correspond to the projected downside risk estimates. The same applies to forecasting, but validation is done between out-of-sample and in-sample downside risk measures.

Panel A in Table 4 shows the back-testing results of the models, while Panel B refers to forecasting. The findings indicate that very few models are adequate at 1 % probability. In the short time-horizon, the CVaR portfolios with precious and industrial metals have relatively high probabilities, 0.575 vs 0.235, which indicates that the model with precious metal is better. On the other hand, mCVaR risk measure is not good in the extreme risk evaluation in short term, taking into account both precious and industrial metals portfolios.

Table 4
Back-testing and forecasting in the optimal portfolios.

	Portfolios with precious metals						Portfolios with industrial metals					
	Raw data		D5		D7		Raw data		D5		D7	
	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR
Panel A. Back-testing												
N	17	0	8	7	0	0	19	3	9	11	54	42
Z-score	0.561	−3.873	−1.787	−2.047	−3.873	−3.873	1.188	−3.035	−1.145	−0.924	10.425	7.258
Prob.	0.575	0.000	0.074	0.041	0.000	0.000	0.235	0.002	0.147	0.356	0.000	0.000
Panel B. Forecasting												
N	3	0	2	9	0	0	26	11	6	4	0	0
Z-score	−1.041	−2.338	−1.473	1.551	−2.338	−2.338	8.897	2.522	0.329	−0.548	−2.303	−2.303
Prob.	0.298	0.019	0.141	0.121	0.019	0.019	0.000	0.012	0.742	0.583	0.021	0.021

Notes: N is the number of failures. Greyed values indicate the highest probability and the best model in terms of adequacy.

In the midterm horizon, the industrial metals portfolio indicates better accuracy when the targeting goal is mCVaR. In the long-term horizon, none of the portfolios is good because the number of empirical failures significantly overstate or understate the number of predicting failures. This means that neither the two-moment CVaR nor the four-moment mCVaR are good risk measures for long-term portfolio investments at very high probability. This calls for model recalibration with other theoretical distributions when long-term investors are at stake.

From the aspect of forecasting, we refer to Su et al. (2023) and use the first 2/3 of the sample for the ex-ante estimation, while the last 1/3 of the sample is used for the ex-post estimation. In particular, the first subset of the sample covers the period from January 2017 to October 2020, while the second subset encompasses the period from November 2020 to October 2022. As for the raw data results, only the CVaR portfolio with gold is good, while portfolios with mCVaR or industrial metals are very inaccurate. In the midterm horizon, CVaR portfolio with industrial metals has the advantage, while none of the portfolios are good when it comes to the long-term horizon.

5.2. Suboptimal portfolio construction with oil constraint

Previous section designed the optimal portfolios, i.e. without imposing any weight-constraints, but in seven out of twelve cases, the share of Brent oil is zero, while in other five cases it is very small. This happens because Brent oil is the riskiest asset in the portfolios, which results in exclusion of Brent oil from the portfolios. As a consequence, this opens a practical hedging problem for oil producers or traders who hold significant quantities of oil. Therefore, this section repeats portfolio optimization imposing the oil constraint of 50 % to all portfolios. Table 5 contains the structure of the oil-dominated portfolios (ODP).

Looking at the portfolios with precious metals, it can be seen that only oil and gold take part in the portfolios in all six cases. This happens because gold has the best characteristics in terms of variance, skewness and kurtosis in all the time-horizons, compared to other precious metals, which means that other metals have no chance of finding their place in the portfolios. These results coincide with the paper of Živkov et al. (2022), which imposed oil constraints in amount of 30 % and 70 % in the minimum variance portfolio with precious metals. In both cases, gold drove all other precious metals out of the portfolio because gold has the lowest variance.

On the other hand, in the investment metal portfolios, the two-asset portfolio are found only in the case when the target is minimum mCVaR, while in the CVaR portfolios, the structure is more heterogeneous. In particular, aluminium has 30 % in short term, while copper and zinc follow with 12 % and 8 %, respectively. Aluminium has the highest share in all time-horizons in the CVaR portfolios because it has relatively low standard deviation and also the second-lowest pairwise correlation with Brent. Therefore, it seems that relatively low correlation between aluminium and Brent plays a decisive factor why aluminium has the highest share in the portfolios with the constraint. By all accounts, covariance matrix has more important role than descriptive statistics is shaping 50 % oil portfolios.

As for the mCVaR portfolios, aluminium has 50 % share in the short-term and midterm horizons because it has relatively favourable third and fourth moment properties. However, in the long-term horizon, somewhat surprisingly, nickel takes 50 % in the mCVaR portfolio, although nickel does not have the best descriptive statistics, particularly in terms of the second and fourth moments. However, nickel has by far the lowest correlation with Brent (0.215) in the long-term horizon, which pushes nickel at the top in the

Table 5
Structure of oil-dominated portfolios in different frequency scales.

	Raw data		D5		D7	
	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR
Panel A: Portfolios with precious metals						
Brent oil	50 %	50 %	50 %	50 %	50 %	50 %
Gold	50 %	50 %	50 %	50 %	50 %	50 %
Silver	0 %	0 %	0 %	0 %	0 %	0 %
Platinum	0 %	0 %	0 %	0 %	0 %	0 %
Palladium	0 %	0 %	0 %	0 %	0 %	0 %
Panel B: Performance of portfolios with precious metals						
Risk measure	−1.669	−7.249	−0.290	−0.820	−0.178	−0.296
HEI	0.456	0.534	0.419	0.414	0.496	0.506
Panel C: Portfolios with industrial metals						
Brent oil	50 %	50 %	50 %	50 %	50 %	50 %
Aluminium	30 %	50 %	43 %	50 %	23 %	0 %
Copper	12 %	0 %	0 %	0 %	0 %	0 %
Nickel	0 %	0 %	0 %	0 %	5 %	50 %
Zinc	8 %	0 %	7 %	0 %	2 %	0 %
Panel D: Performance of portfolios with industrial metals						
Risk measure	−1.818	−5.935	−0.307	−0.541	−0.228	−0.264
HEI	0.408	0.609	0.384	0.587	0.367	0.524

Note: Greyed values indicate higher HEI, comparing the precious and industrial metals portfolios.

long-term mCVaR portfolio. This is yet another evidence that covariance matrix has a crucial role in moulding the portfolio with the oil constraint.

Comparing risk performances of the two ODPs, it can be seen that precious metals portfolio is better when the target is CVaR, while industrial metals have upper hand when mCVaR is the optimized function.

As in the case of the optimal portfolios, ODPs also need to be assessed from the aspects of back-testing and forecasting, so it can be decided which portfolio is better. Table 6 presents the results of the in-sample back-testing and out-of-sample forecasting. According to the results, ODPs with gold in the short-term and midterm horizons are better at verifying whether realized losses match projected CVaR estimates. Comparing the number of violations between out-of-sample and in-sample, ODPs with gold are also better. Looking at Tables 5 and 6, it seem that the CVaR portfolios with gold in the short- and midterm horizons are the winning strategy for agents who works with Brent oil.

ODPs are riskier compared with the optimal portfolio, which is expected, but they are more pragmatic and intended for practical use. Besides, finding gold as the only auxiliary instrument in ODPs carries two additional benefits for portfolio investors. First, transaction costs are lower when fewer instruments enter the portfolio. And second, the gold market is the most liquid market of all analysed metals, as it has the largest daily trading volume (see Table 7), so building a portfolio with gold is the easiest because it is the most accessible.

6. Complementary analyses

In order to be more thorough and comprehensive in the research, two additional calculations are carried out. First, we want to check what the hedging result would be if WTI oil is considered instead of Brent oil. And second, we explore the implications when the US 10Y bonds are included in the portfolios.

6.1. Portfolio construction with WTI oil

Brent and West Texas Intermediate (WTI) are the two main benchmark oils. Brent is a weighted average of oil prices from the different OPEC members around the world, while WTI is the benchmark for the U.S. light oil market. These oil prices are heavily correlated, but not identical. WTI has a lower sulphur content than Brent, so it is chipper to process and refine. This is one of the reasons why WTI crude usually has lower price than Brent. Other factors, such as supply and demand, production interruptions and geopolitical influences, also play a role in shaping the Brent/WTI spread.

Since Brent and WTI prices diverge, it is interesting to see if and to what extent the precious and industrial metals portfolios differ when WTI is hedged. Table 8 shows the structures of the portfolios with WTI, and it can be seen that there are very small differences between Tables 3 and 8, which is expected, taking into account that the two prices have very similar dynamics. WTI is slightly riskier than Brent, which translates to a lower share of WTI in Table 8 compared to Brent in Table 3. For instance, in the raw CVaR portfolio with precious metals, the share of WTI is 5 %, while Brent is 8 %. In the mCVaR portfolio, the relation is 4 % vs 8 %. As for the raw CVaR portfolio with industrial metals, the share of WTI is 3 % *vis-à-vis* 5 % of Brent, while no difference is found in the mCVaR portfolio. In the D5 and D7 portfolios, the changes are very small or even non-existent. The tiny changes recorded in the auxiliary assets between Tables 3 and 8 occur due to the difference in the length of the time series which is a consequence of the synchronization. The level of downside risk is slightly higher in the WTI portfolios because WTI is riskier than Brent.

The back-testing and forecasting results are very similar with the Brent-portfolios, so they are available upon request due to space parsimony.

6.2. Portfolio construction with the US 10Y bonds

This subsection explores the implications if some unrelated assets, such as the US 10Y bonds, are added to the portfolios. A lower

Table 6
Back-testing and forecasting in the oil-dominated portfolios.

	Portfolios with precious metals						Portfolios with industrial metals					
	Raw data		D5		D7		Raw data		D5		D7	
	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR
Panel A. Back-testing												
N	19	0	17	0	34	0	21	2	12	6	34	0
Z-score	1.082	−3.873	0.561	−3.873	4.994	−3.873	1.716	−3.299	−0.660	−2.243	5.147	−3.827
Prob.	0.279	0.000	0.575	0.000	0.000	0.000	0.086	0.001	0.509	0.025	0.000	0.000
Panel B. Forecasting												
N	5	0	3	0	87	97	9	0	0	0	50	25
Z-score	−0.177	−2.338	−1.041	−2.338	35.255	39.576	1.645	−2.303	−2.303	−2.303	19.629	8.663
Prob.	0.859	0.019	0.298	0.019	0.000	0.000	0.100	0.021	0.021	0.021	0.000	0.000

Notes: N is the number of failures. Greyed values indicate the highest probability and the best model in terms of adequacy.

Table 7
Average daily trading volumes in the futures markets in 2022.

	Gold	Silver	Platinum	Palladium	Aluminium	Copper	Nickel	Zinc
Volume	180,122	55,927	5,844	213	38,719	15,677	8,447	19,140

Source: investing.com website.

Table 8
Structure of the optimal portfolios with WTI in different frequency scales.

	Raw data		D5		D7	
	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR
Panel A: Portfolios with precious metals						
WTI oil	5 %	4 %	2 %	0 %	0 %	0 %
Gold	94 %	96 %	98 %	100 %	78 %	71 %
Silver	0 %	0 %	0 %	0 %	0 %	0 %
Platinum	0 %	0 %	0 %	0 %	22 %	29 %
Palladium	1 %	0 %	0 %	0 %	0 %	0 %
Panel B: Performance of portfolios with precious metals						
Risk measure	−1.028	−2.150	−0.176	−0.178	−0.073	−0.059
HEI	0.714	0.901	0.698	0.886	0.833	0.922
Panel C: Portfolios with industrial metals						
WTI oil	3 %	0 %	4 %	0 %	0 %	0 %
Aluminium	43 %	39 %	71 %	59 %	34 %	7 %
Copper	39 %	42 %	25 %	29 %	49 %	92 %
Nickel	1 %	0 %	0 %	11 %	0 %	0 %
Zinc	14 %	19 %	0 %	1 %	17 %	1 %
Panel D: Performance of portfolios with industrial metals						
Risk measure	−1.243	−1.639	−0.230	−0.223	−0.134	−0.152
HEI	0.657	0.926	0.621	0.861	0.688	0.780

Note: Greyed values indicate higher HEI, comparing the precious and industrial metals portfolios.

correlation increases the effectiveness of diversification, and Table 9 shows that the multiscale average correlation between the bonds and assets is very low. This gives enough reason to believe that adding bonds to the portfolios would improve its performance. Table 10 presents the constructed six-asset portfolios and their risk performances when the bonds are included in the portfolios.

Comparing Tables 10 and 3, it can be seen that the presence of the bonds affects the share of other assets in the portfolios. For example, the weighting of Brent and gold is reduced in the portfolios when the bonds are included. The same applies for the industrial metal portfolios, where can be seen that the shares of industrial metals and Brent are slightly lower. However, an important implication is that all HEI indices are little bit higher in Table 10, than in Table 3. This suggests that adding long-term bonds to the portfolios slightly improves hedging performance. However, this raises the question of a trade-off between slightly better hedging and higher trading costs.

Comparing the bond and non-bond risk hedging performance between precious and industrial metals portfolios, the findings do not change because in most cases the precious metal portfolios have better hedging results. Back-testing and forecasting of the portfolios with bonds do not change significantly compared to the non-bonds portfolios. These results are available upon request due to space brevity.

7. Conclusion

This study combines Brent oil with precious and industrial metals in an effort to reduce extreme risk of Brent oil in the short-, mid- and long-term horizons. Before portfolio optimization process, all empirical time-series are transformed into D5 and D7 wavelet details, which represent the midterm and long-term horizons. Raw data is used for the short-term horizon. Extreme risk of the optimized portfolios is measured by the parametric and semiparametric CVaR metrics. Besides, we also construct oil-dominated portfolios with 50 % Brent oil constraint, in order to overcome possible oil exclusion from the portfolios, because oil is the riskiest asset in the portfolios. Back-testing and forecasting procedures are employed to verify adequacy of the created portfolios. Two complementary analyses are also performed.

Several findings are worth mentioning. First, based on the risk-characteristics of the portfolios, the optimal portfolios with gold have the lowest downside risk in most cases. In the CVaR portfolio, this happens because gold has the lowest standard deviation, while in the mCVaR portfolio this is because gold reports the lowest skewness and kurtosis. Gold also has the lowest pairwise correlation with oil, which makes gold the most favourable precious metal to combine with Brent. Only the raw data mCVaR portfolio with industrial

Table 9
Average correlation between the US 10Y bond and the assets in the two portfolios.

	Portfolio with precious metals			Portfolio with industrial metals		
	Raw data	D5	D7	Raw data	D5	D7
Correlation	0.009	−0.034	0.327	0.119	0.071	0.451

Table 10
Structure of the optimal portfolios with US 10Y bonds.

	Raw data		D5		D7	
	CVaR	mCVaR	CVaR	mCVaR	CVaR	mCVaR
Panel A: Portfolios with precious metals						
Brent oil	3 %	7 %	0 %	0 %	0 %	0 %
Gold	88 %	91 %	82 %	94 %	81 %	77 %
Silver	0 %	0 %	0 %	0 %	0 %	0 %
Platinum	0 %	0 %	0 %	0 %	14 %	20 %
Palladium	0 %	0 %	0 %	0 %	0 %	0 %
US 10Y bond	9 %	2 %	18 %	6 %	5 %	3 %
Panel B: Performance of portfolios with precious metals						
Risk measure	−0.947	−2.035	−0.145	−0.171	−0.072	−0.059
HEI	0.692	0.863	0.710	0.878	0.795	0.902
Panel C: Portfolios with industrial metals						
Brent oil	3 %	0 %	2 %	1 %	0 %	0 %
Aluminium	42 %	37 %	61 %	46 %	32 %	0 %
Copper	36 %	40 %	18 %	22 %	51 %	23 %
Nickel	1 %	0 %	2 %	13 %	0 %	0 %
Zinc	14 %	18 %	0 %	0 %	17 %	77 %
US 10Y bond	4 %	5 %	17 %	18 %	0 %	0 %
Panel D: Performance of portfolios with industrial metals						
Risk measure	−1.230	−1.600	−0.210	−0.188	−0.136	−0.149
HEI	0.599	0.894	0.581	0.855	0.623	0.732

Note: Greyed values indicate higher HEI, comparing the precious and industrial metals portfolios.

metals has better hedging result than the precious metals portfolio. Aluminium and copper have the highest share in both CVaR and mCVaR portfolios because these two have the lowest standard deviation and also very good third and fourth moment properties.

As for the back-testing results, the raw data CVaR portfolio with precious metals is the best, while the D5 mCVaR portfolio with industrial metals has upper hand. In the long-term horizon, none of the portfolios is good, which means that CVaR and mCVaR models are not adequate risk functions for identifying realized risk in the long term. In terms of forecasting, CVaR portfolios with precious and industrial metals are the best in the short- and midterm horizons, respectively, while neither portfolio is good for predicting long-term investments.

In the case of ODPs, it turns out that the CVaR portfolios with gold are the best in terms of the lowest risk as well as back-testing and forecasting performances. This is true for the short and medium term horizons, while in the long run none of the portfolios is adequate.

When Brent is replaced by WTI crude in the optimization process, very similar results are obtained, but the share of WTI is somewhat smaller in the portfolios because WTI has higher risk. Adding the US 10Y bond to the portfolios reduces the risk of the portfolios because the bonds are very weakly correlated with the commodities.

This paper is the first one that examines the role of precious and industrial metals as hedgers of Brent oil in multi-frequency portfolios with different complex risk measures. Although mCVaR is a more elaborate risk algorithm, the paper shows that the CVaR function is a better risk measure than mCVaR in most cases, because it is better at back-testing and forecasting realized extreme risk. According to the results, both group of metals significantly reduce extreme risk of Brent oil, but gold proved to be the best hedging asset for most of the time in the optimal and ODP portfolios due to its traditionally low risk. Therefore, this paper could provide guidance to oil market participants on how to construct their risk-minimizing portfolios, taking into account the different time horizons in which they operate.

CRediT authorship contribution statement

Dejan Živkov: Conceptualization, Data curation, Investigation, Methodology, Software, Writing – original draft. **Slavica Manić:** Conceptualization, Formal analysis, Methodology, Writing – review & editing. **Marina Gajić-Glamočlija:** Conceptualization, Formal analysis, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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