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SUBLIMATION OF VARIOUS MODELS RESULTS OF MULTI-CRITERIA ANALYSIS AS A FUNCTION OF IMPROVEMENT OF ALTERNATIVE RANK RELEVANCE

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Abstract

The selection of models for ranking and evaluation of solution variants (alternatives) is a multi-criteria problem, similar to the problems which are solved by its application. Since there is no ideal solution, but only an optimal one, it is clear that any chosen model will have certain, more or less extensive, drawbacks. In addition, results obtained by using various models are presented in different ways and are in different forms. Also, there is always a question of obtained results relevance due to the relativity of selected evaluation criteria. In this paper, a seemingly simple model is presented, which uses magnitude normalization of results obtained using at least three models of multi-criteria rank alternatives, and reduces them to comparable dimensions, and then determines the common result of all processes by sublimation, which greatly increases its relevance and reliability.

Keywords: Magnitude normalization, ranking of alternatives, decision tree, multi-criteria decision making,

1. Introduction

The problems related to determining of an optimal solution, i.e. optimisation taska, are frequently encountered and solved in everyday life. They are encountered almost everywhere, in technical and economic systems, in families, companies, sport temas, etc, [1]. The decision making process and the choice of „best“ alternative is typically based on more than one criteria and a number of limitations.

In all tasks involved in this process, the implicit tendency of a man to find a solution which satisfies his needs as much as possible, i.e. which is most beneficial, without breaking of texisting limitations.

Asuming that the multi-criteria desicion problem is solved using an analytical approach, one can also asume that a decision space (alternatives, actions) also exists, with an introduced variable A_x and the space of possible results, C_q. In case both spaces are normed, with an allowed mapping function, $f : A_x \rightarrow C_q$, the following set of solutions is achieved, $q_0 = f(x_0) \subset C_q$, where $x_0 \subset A_x$ represents the set of acceptable decisions, [2].

Based on this it can be concluded that, unfortunately, such problems (known as multi-criteria optimisation) do not have a unique and global solution, i.e. there is no alternative solution which is optimal for all criteria simultaneously. All of this indicates that a final unique solution cannot be determined without involving the decision maker. The decision maker needs to adopt a solution at the end of the process. The solution accepted by the decision maker is referred to as best or preferred solution.The task of multi-criteria optimisation, i.e. analysts who perform it, is to aid the decision maker in choosing the solution which is considered as best in given conditions.

The basic question related to multi-criteria decision making issues involves the determining of procedures for the selection of decisions which correspond to the desired solution, with the possibility of selecting and singling out (a very common practice) of the most acceptable alternative.

Today, there is a great number of generally accepted methods and models of multi-criteria optimisation. Thus, the selection of evaluation models and solution (alternatives) variant ranking is a multi-criteria problem, like the problem it is solving. Since there is no ideal solution, but only an optimal one, it is clear that any chosen model will have certain, more or less extensive, drawbacks.

Each model treats four basic components of decision making process in a different way, and these components include: the goal, attributes, alternatives and decision making preferences. Hence, output results are often highly heterogeneous and typically expressed in form of greatly varying records.

It is not uncommon to consider the question of relevance, for both the selected criteria and the way in which the decision maker's preferences are treated, along with the relevance of the way in which the MCA method is selected.

With the idea of contributing to relevance and credibility of the MCA process and the generated solutions alike, suggested in this paper is a model, which, using result magnitude normalisation, obtained by using at least three models of multi-criteria alternative ranking, chosen in a scientifically acceptable and intelligent manner, provides comparable dimensions, and through their sublimation, determines the common, relevant and credible process for result evaluation and alternative ranking.

2. Method

"Multiple-criteria decision aid" (MCDA) is the name of a scientific field which deals with the development of methodologies and methods that are used to aid decision makers in complex situations which involve multiple conflicting goals, i.e. criteria.

Multi-criteria decision making is an area which has gained a lot of significance in the past twenty years, taking into account that every decision making process involves considering of numerous criteria, which are often in conflict with each other or are expressed using different measuring units which cannot be mutually compared.

From the sixties until now, a large number of multi-criteria analysis methods were developed, and can be classified in multiple ways [3]. The most well-known MCA method classification, [4], distinguishes between different methods according to their type and relevant characteristics of information provided by the decision maker (Table 1).

Table 1. Basic classification of MCA methods

Methods without attribute information	Methods which require certain attribute information
Domination method MAXIMIN method MAXIMAX method	Conjunctive method Disjunctive method Lexicographic method Linear assigning method Simple additive weight method AHP method ANP method ELECTRE TOPSIS SAW PROMETHEE

Analytical hierarchy process (AHP) belongs to the class of "soft" optimisation methods. Basically, it represents a specific tool for forming and analysing of decision hierarchies. Above all, AHP enables interactive creation of problem hierarchy as a means for preparing of the decision making scenario, followed by evaluation of pairs of hierarchy elements (goals, criteria (sub-criteria), and alternative) in a top-down direction. Finally, a synthesis of all evaluations is performed, and weight coefficients for all hierarchy elements are determined, using a strictly determined mathematical model. The sum of weight coefficients of elements for all hierarchy levels equals one (1), which allows the decision maker to rank all elements in both horizontal and vertical sense. AHP enables an interactive analysis of evaluation procedure sensitivity to final ranks of hierarchy elements. In addition, during hierarchy elements evaluation, until the procedure ends and the results are synthesised, resonating consistency of the decision maker is checked and the accuracy of obtained alternative rankings and criteria is determined.

Observed from a methodological standpoint, AHP represents a multi-criteria technique which is based on decomposing complex problems into a hierarchy. The goal is located at the top of the hierarchy, whereas criteria, sub-criteria and alternatives are at lower levels. Shown as an illustration (Figure 1), hierarchy is made of a goal, three criteria and four alternatives. Hierarchy does not need to be complete.

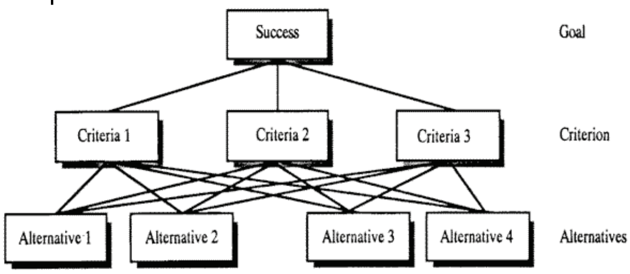

Figure 1. AHP hierarchy

AHP is a flexible method, since it enables a relatively simple way of determining relations between influencing factors, recognising their explicit or relative influence and the significance in realistic conditions and determining of dominance of one factor relative to the other, in case of complex problems with multiple criteria and alternatives. The method, namely, anticipates the fact that even the most complex problem can be decomposed into a hierarchy in a way that ensures that further analysis involves both qualitative and quantitative aspects of the problem. AHP keeps all parts of the hierarchy connected, thus it is simple to observe how a change in one factor influences other factors. Problem complexity increases with the number of criteria and alternatives. The ability of a human mind to distinguish between a large number of alternatives and criteria is limited and thus, when forming a hierarchy it is not recommended to use more than 5±2 elements on the same level [5]. The application of the AHP method can be explained in four basic steps [6]:

- Development of a hierarchy model of the decision making problem, with its goal on top, criteria and sub-criteria on lower, and alternatives at the bottom level of the model (Figure 1).
- Comparison of pairs of elements on every level of a hierarchy structure, wherein decision maker's preferences are expressed using the Saaty scale of relative relevance, which has 5 stages and 4 interstages of intensity, which are verbally described and have corresponding numerical values ranging from 1-9, [5], [6],
- Calculating of local priorities (weights) for criteria, sub-criteria and alternatives based on evaluation of relative relevance of elements of the correspon–ding hierarchy structure level, using a mathematical model, which are then synthesised into total alter–native priorities. Total priority of an individual alter–native is calculated by summing of its local priorities pondered with higher level element weights.In the end, a sensitivity analysis is performed.

The result of an AHP analysis is a ranking list of all considered alternatives, wherein the alternative with the highest relative weight is located at the top of the list.

SAW method (Simple Additive Weighting Method) represents one of the most well-known and frequently used multi-criteria decision making methods. In addition to providing a very simple and practical procedure of ranking alternatives, the results obtained by its application do not deviate from the results obtained by some of the so-called advanced methods. It is applied directly to the decision making matrix and consists of three steps:

- Decision matrix normalisation;
- Multiplying of a normalised matrix using pondered coefficients, and
- Summing of "weighted" parameters for each alternative.

The method is particularly convenient when criteria are of the same or similar nature. The decision maker should assign each criteria a corresponding weight or pondered coefficient (wk, k = 1, 2, ..., m). The best value of an alternative is the one where the sum of "weighted" parameters is the highest.

PROMETHEE method (Preference Ranking Organization Method for Enrichment Evaluations) belongs to a group of multi-criteria choice methods within a set of alternatives described by multiple attributes which are used as criteria. This method enables the aggregation of qualitative and quantitative criteria of varying relevance into a relation of partial arrangement in a set of alternatives (PROMETHEE I) or into a unique score (PROMETHEE II), based on alternatives that can be fully ranked.

The application of the PROMETHEE method is characterised by two following steps:

- Construction of preference relation in a set of alternatives A,
- use of this relation in order to respond to a given problem.

During the first step, a complex preference relation is formed to emphasize the fact that this relation is based on acknowledging multiple criteria (this relation is originally called the outranking relation), based on generalisation of the term criteria. The preference index is defined and a complex preference relation is obtained, which is expressed using a preference graph. The essence of this step is that the decision makers needs to state their preferences between two alternatives (actions, activities) in accordance with each criteria, based on the difference between criteria values being compared. Preference relation created in this way is used so that input and output of each flow in the graph are calculated. Based on these flows, the decision maker can introduce a partial arrangement (PROMETHEE I) or a total arrangement (PROMETHEE II) into the set of alternatives.

An analyst or DSS (Decision Support System) gives the decision maker a possibility to choose a solution using an appropriate algorithm or method, based on the information given by the decision maker. Once all relevant data are obtained from the decision maker, the system suggests a solution.

The analyst or DSS suggest that solution which is ranked first, according to the selected method, i.e. the one that scored the highest number of "points", in other words, has the highest relative weight.

Diversity of methods is not only reflected in the way the basic decision making parameters are treated, but also in the way in which obtained results are presented. Whereas in case of some method the results are expressed in integers, in other cases they are expressed in percents or are shown on a 0-1 scale.

The Decision Tree is a graphically presented decision making technique, which involves a set of connected branches, where each branch represents a decision alternative or state. According to the common convention, the node represented by a square represents the decision alternative (decision node), whereas a circle represents a state (possibility node).

Based on the characteristics of each MCA method, a decision tree was formed in order to select the appropriate method during the solving of a specific multi-criteria problem. Shown in Figure 2 is the decision tree used for selecting of an MCA method, in accordance with the problem being solved.

Considering that the suggested model for selection of an optimal variant of corridors in linear infrastructural objects requires them to be explicitly defined, and that their complete order needs to be defined as well, wherein the decision making matrix is not defined, which should be done during the decision making process, the decision tree decided on selecting the AHP method. The selection path is denoted by the red lines.

The normalisation is a procedure during which physical quantities in expressions are represented by unnamed numbers which in turn are obtained by dividing a given physical quantity by a selected base quantity. Base quantities are chosen to have the same physical dimension as the quantity they are normalising, so that normalisation makes sense, e.g. simplification of expression writing.

By performing normalisation, the values of quantities are equalised. Normalisation can be twofold:

Vector normalisation - each row-vector of decision making is divided by its norm, wherein the normalised value n_{ij}, of the normalised decision making matrix N, for *max* type criteria, is obtained by the following expression:

$$n_{ij} = \frac{f_{ij}}{\left(\sum_{i=1}^{m} f_{ij}^2\right)^{1/2}},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \qquad (1)$$

whereas in case of *min* type criteria, the following expression is used:

$$n_{ij} = 1 - \frac{f_{ij}}{\left(\sum_{i=1}^{m} f_{ij}^2\right)^{1/2}},\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \qquad (2)$$

The advantage of this transformation lies in the fact that all criteria can be expressed using measures that have their own units.

The linear scale - output (result) of a criteria is divided by its maximum value, and then the transformed output is determined based on the following expression:

$$l_{ij} = \frac{f_{ij}}{f_j^*} = \frac{f_{ij}}{\max f_{ij}},\ f_j^* = \left\{ f_i \,\middle|\, \max_i f_{ij} \right\}, \qquad (3)$$

$$i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

Values lij are within the interval (0,1) and results are that approach the value 1 are more favourable.

In the case of type min criteria, the element of linearised decision making matrix is determined in the following way:

$$l_{ij} = \frac{f_j^{\min}}{f_{ij}} = \frac{\min f_j}{f_{ij}}, f_j^{\min} = \left\{ f_j \,\middle|\, \min_i f_{ij} \right\}, \qquad (4)$$

$$i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

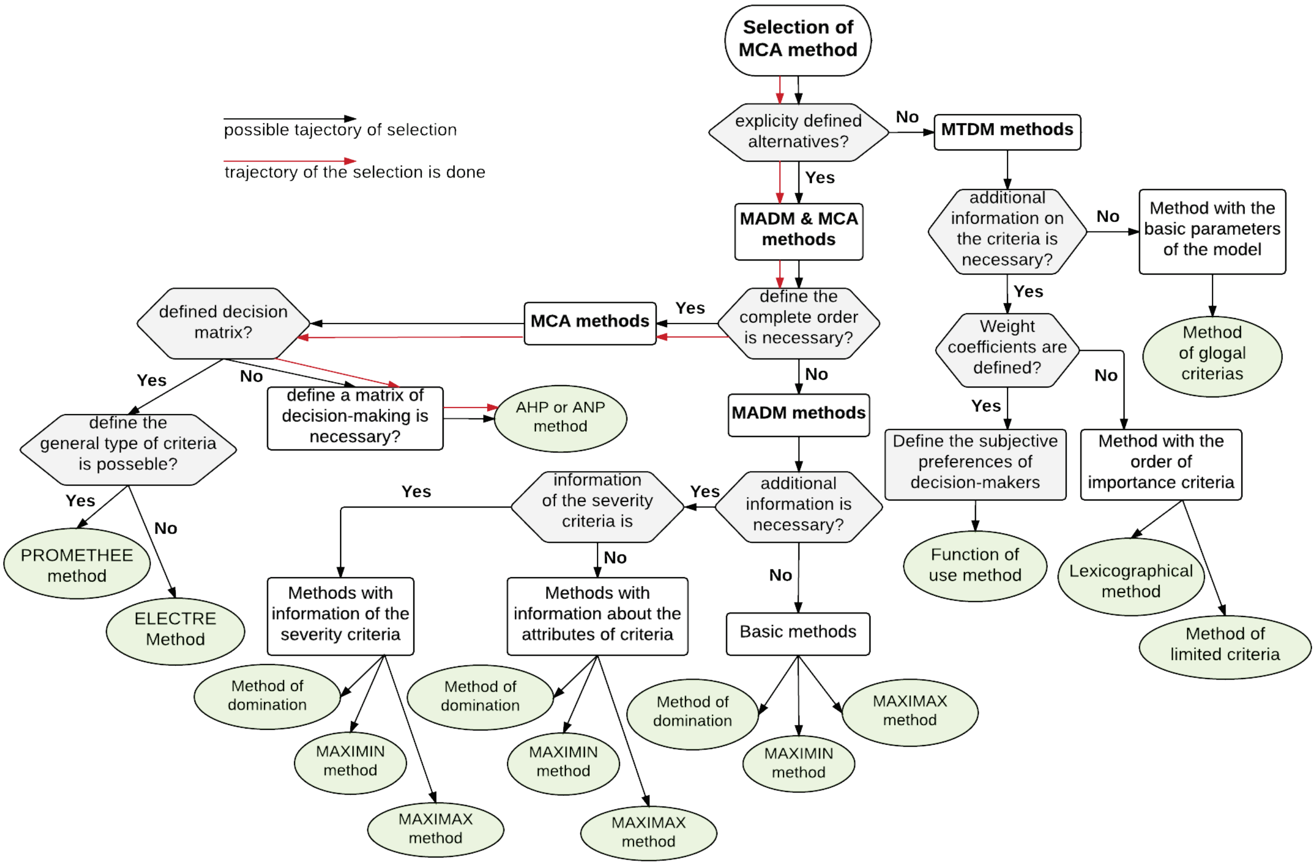

Figure 2. Decision tree for selection of VKA method

3. Results

For the purpose of this research, data from a doctoral dissertation written by one of the authors, [7], in which a complex decision making model for the process of selecting of an optimal urban solution of a corridor for a linear infrastructural object was developed and tested (3N-AHP model). This model was based on the methodology of multi-actor multi-criteria analysis (MAMCA) methodology, [8], and the application of "soft" optimisation AHP, integrated with other scientific methods (stakeholder analysis, survey, descriptive statistics, factor analysis).

A decision tree was used to determine the AHP method as the most adequate method for evaluation and ranking of alternatives in such cases.

However, due to the diversity of infrastructural objects and characteristics of individual problems that occur during their design, certain "detours" in decision tree nodes are possible; hence the line of selection could lead to some other MCA method.

In this case, the SAW and PROMETHEE methods were located at the end of the path. For the purpose of this paper, a comparative analysis was performed, along with sublimation and result normalisation, based on applying MCA to each of the three models individually.

Since different models were used, final results are presented in different ways (Table 2).

Table 2. Values of alternative priorities based on three evaluation models

ALTERNATIVE	SAW	PROMET.	AHP
West	0,58676	35	0,200
Centre	0,89568	45	0,261
East 1	0,88902	52	0,274
East 2	0,86347	53	0,265

In order to perform mutual comparison of results, the alternative priority values obtained from SAW and PROMETHEE methods were normalised, and in this way values shown on a scale of 0-1 were obtained.

Shown in Table 2 are the output values obtained in a way during the model application, whereas Table 3 shows the normalised values on a scale of 0-1 for results obtained by applying all three models.

Given in Table 4 are the sums of alternative priority values obtained by simple summing of individual priorities from Table 3, for each alternative.

Table 3. Normalised alternative priority values on a scale of 0-1

ALTERNATIVE	SAW	PROMET.	AHP
West	0,181	0,189	0,200
Centre	0,277	0,243	0,261
East 1	0,275	0,281	0,274
East 2	0,267	0,286	0,265

Table 4. Sum values of alternative priority values based on all three models

ALTERNATIVE	TOTAL PRIORITY
West	0,570
Centre	0,781
East 1	0,830
East 2	0,818

Given in Table 5 are the normalised alternative priority vaues based on the three evaluation models, and based on them, ranking of alternative was performed. Based on the summed results of evaluation obtained from all three models, the best ranked alternative, among the four taken into consideration, was West one with a rating of 0.277. Alternative East had the lowest rank.

Table 5. Final ranking of alternatives based on normalised priority values

ALTERNATIVE	PRIORITY	RANK
West	0,277	1
Centre	0,273	2
East 1	0,260	3
East 2	0,190	4

Results obtained in this way represent a sublimation of individual results of implementing each of the three models, and thereby methodologies, the ways of treating all elements of decision making, along with various decision maker preferences. Such results greatly contribute to the reconciliation of attitudes that interested parties and decision makers have, since generated solutions are made more credible and relevant, i.e. more acceptable for a larger number of them.

4. Discussion

A great number of MCA methods, which represent their output data (results) in various ways (Table 2), largely contribute to the poor understanding that end users have for them. In additions, the diversity of methodologies which are used in MCA resulted in divided opinions among the interested parties regarding which method has a better, more acceptable way of treating their preferences in terms of priorities related to both evaluation criteria and possible alternative solutions of problems.

Thus, a question of which method should be used and when during the decision making process often arises.

The offered model, even though seemingly simple, aids the analyst in selecting the adequate MCA method(s), using a decision tree and based on the characteristics of the problem. If due to the nature of the problem being solved, the path unambiguously and exclusively leads to a single method, then the suggested model implies mutual comparison and result sublimation, i.e. the generating of a single common solution for all MCA models that were used. Through this process all results, with the aid of the value normalisation, are reduced to homogeneous and comparable quantities. This contributes to their easier understanding, as well as easier operating in terms of mathematics. Once the process is complete, a unique (common) solution for all models used is generated.

Results obtained in this way are a sublimation of individual results of the implementation of each of the three models, and thereby the methodologies, ways of treating all decision making elements, along with different decision maker priorities. Such results greatly contribute to the reconciliation of attitudes that interested parties and decision makers have, since generated solutions are made more credible and relevant, i.e. more acceptable for a larger number of them.

5. Conclusion

The selection of a model for ranking and evaluating of solution variants (alternatives) is, similar to the problems it is solving, a multi-criteria problem.

The suggested model, by using a decision tree and the intelligent selection method, aids the analyst in reliably selecting an adequate MCA method (or more methods).

In case the number of used methods is greater than one, the suggested model, with the aid of result magnitude normalisation, which reduces heterogeneous quantities to homogeneous ones, makes results from different models easy to compare.

The possibility of result sublimation into a common result, which does not only sublime values, but also the methodology, interested parties and decision maker preferences, contributes to the reconciliation of attitudes that interested parties and decision makers have, since generated solutions are made more credible and relevant, i.e. more acceptable for a larger number of them.

Further research should focus on defining even more reliable methods of result sublimation, which would greatly contribute to their relevance.

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