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A NOTE ON DICTATORSHIP, LIBERALISM AND THE PARETO RULE

ABSTRACT: *This note presents a short comment and explanation of the approach and results presented in my previous papers published in this journal. I have divided the traditional social choice axioms, introduced by Kenneth Arrow and Amartya Sen, into two classes, based on their linguistic and mathematical complexity. The first class consists of 'the unrestricted domain' and 'the independence of irrelevant alternatives', (Arrow, 1963; (Sen, 1970b;Maskin, 2020), which need a higher-order language, and can be treated as meta-axioms. The second class contains a group of linguistically simpler axioms, such as 'dictatorship', 'liberalism' and 'the Pareto rule'. Naturally, it is possible to make an easier logical analysis of the deductive properties and relationships between the axioms belonging to the second class, and the paper explains a method for their simplification. The basic conclusion is that after these simplifications, we obtain a fragment of the traditional Arrow-Sen theory in which we can also prove well-known impossibilities, including the counterparts of Arrow's and Sen's theorems. I consider that the value of each simplified approach lies in providing an opportunity to a wider circle of readers to better understand the basic ideas, results and spirit of traditional Social Choice Theory.*

KEY WORDS: *impossibility theorems; social choice theory; dictatorship; liberalism; Pareto rule.*

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Elements of traditional Social Choice Theory relevant to this note are given in Fishburn (1973) and Sen (1970a, 1970b, 1995), and a deep analysis of the language and logic of this theory was presented in Routhley (1979).

The results presented in Boričić (2009, 2014a, 2014b) cannot be considered as a repetition of the well–known theorems of Social Choice Theory, but as their analogues in a new context, subtly simplified and modified, but similar. Here, we want to explain the methodological and logical background of axiomatizations of fragments of traditional Social Choice Theory presented in our previous works Boričić (2009, 2014a, 2014b) or Boričić, & Srećković (2023). Namely, in both papers Boričić (2009, 2014b) the Pareto rule, and the dictatorship and liberalism axioms are given in an essentially different, but very similar and recognizable form. During our research, it was intuitively clear that we obtained some simpler fragments of traditional Social Choice Theory, but their pure logical relationships and methodological argumentations were absent. Now, in hindsight we are able to formally explain the status of these simplified forms of traditional axioms.

The traditional axioms employ quantification over relations and combine natural and higher–order formal languages, such as follows:

The Pareto rule TP, prefixed by **T** to denote a traditional form of **P**, claims that, for **all profiles** $\mathcal{P}$ and all alternatives $x, y \in X$, if every individual $i \in V$ prefers x to y, then society must prefer x to y. This is, in fact, a weak version of the Pareto principle, as introduced by Kenneth Arrow (see Arrow (1963) or Sen (1970a, 1970b, 1995)).

The dictatorship axiom TD states that there is a person $i \in V$, a dictator, having such power that, for **all profiles** $\mathcal{P}$ and all alternatives $x, y \in X$, if i prefers x to y, then society must prefer x to y as well (see Arrow (1963) or Sen (1970a, 1970b)).

The liberalism axiom TL supposes that, for **all profiles** $\mathcal{P}$ and each individual $i \in V$ there is at least one pair of alternatives $(x, y) \in X^2$ such that $x \neq y \wedge (xP_iy \to xPy) \wedge (yP_ix \to yPx)$ (see Sen (1970a, 1970b)).

These conditions can be respectively presented more formally in the following way:

$$\mathbf{TP}: \quad (\forall \mathcal{P})(\forall x, y \in X)((\forall i \in V)xP_iy \to xPy),$$

$$\mathbf{TD}: \quad (\exists i \in V)(\forall \mathcal{P})(\forall x, y \in X)(xP_iy \to xPy)$$

and

$$\mathbf{TL}: \quad (\forall\mathcal{P})(\forall i \in V)(\exists x, y \in X)(x \neq y \wedge (xP_iy \rightarrow xPy) \wedge (yP_ix \rightarrow yPx))$$

Meanwhile, in Boričić (2009, 2014b), we use their simplified variations such as:

$$\mathbf{SP}: \quad (\forall x, y \in X)((\forall i \in V)xP_iy \rightarrow xPy),$$

prefixed by **S** to denote a simplified form of **P**,

$$\mathbf{SD}: \quad (\exists i \in V)(\forall x, y \in X)(xP_iy \rightarrow xPy)$$

and

$$\mathbf{SL}: (\forall i \in V)(\exists x, y \in X)(x \neq y \wedge (xP_iy \rightarrow xPy) \wedge (yP_ix \rightarrow yPx))$$

supposing that these variations hold for **all profiles** $\mathcal{P}$, which is in line with the general assumption about the schematic character of axioms.

If we employ the deduction relation $A \vdash B$ to denote that "B can be derived from A", as in Boričić (2009, 2014b), we note that in all cases we have:

$$\mathbf{TP} \vdash \mathbf{SP}, \quad \mathbf{TD} \vdash \mathbf{SD} \ \text{ and } \ \mathbf{TL} \vdash \mathbf{SL}$$

i.e. that each simplified form is deductively entailed by an appropriate traditional form, fact which is based on the following general logical law:

$$\exists x \forall y A \vdash \forall y \exists x A$$

and then moving the universal quantification $\forall y$ to some kind of metatheoretical level. This operation can be of great importance when the object '$\forall y$' belongs essentially to a higher–order language, such as $\forall\mathcal{P}$. By this procedure we can obtain a similar but essentially simpler fragment of the theory which could be more approachable than the original one. Analogue statements were expressed in the first–order language, in Boričić (2009) and Boričić, & Srećković (2023), and in an almost propositional language, in Boričić (2014a, 2014b).

Finally, let us consider an example concerning famous Arrow's impossibility theorem. This theorem can be formulated as **TP** $\vdash$ **TD**, assuming that conditions of 'unrestricted domain' and 'the independence of irrelevant alternatives' hold, in original Arrow's theory, while its analogue, a similar statement, in this new simplified context, is the following one: $\mathbf{SP} \vdash \mathbf{SD}$. Let us emphasize that neither counterpart $\mathbf{SP} \vdash \mathbf{SD}$ implies Arrow's original theorem $\mathbf{TP} \vdash \mathbf{TD}$, nor vice versa. Consequently, these two statements can be considered as two roughly connected facts in two parallel worlds. Similarly, we can present a counterpart of Chichilnisky's original theorem Chichilnisky (1982), 'the impossibility of a non–Paretian dictator': $\mathbf{TD} \vdash \mathbf{TP}$, and its counterpart in our simplified context: $\mathbf{SD} \vdash \mathbf{SP}$, asserting again that there is no immediate formal logical connection between these two statements. But, on the other side, bearing in mind that $\mathbf{TP} \vdash \mathbf{SP}$ and $\mathbf{TL} \vdash \mathbf{SL}$, we can directly derive well–known Sen's 'impossibility of a Paretian liberal': $\mathbf{TP}, \mathbf{TL} \vdash$, from its simplified version $\mathbf{SP}, \mathbf{SL} \vdash$, meaning that the axioms **SP** and **SL**, and, consequently, the axioms **TP** and **TL**, when they appear together, make the theory inconsistent.

I consider that the value of this simplified approach is in giving an opportunity to a wider circle of readers to better understand the basic ideas and results of traditional Social Choice Theory.

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