CONFIDENCE INTERVALS FOR THE POPULATION STANDARD DEVIATION: SIMPLE RANDOM SAMPLING VS. RANKED SET SAMPLING

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Abstract. *Ranked set sampling (RSS) is the cost-efficient sampling procedure. This procedure gives more efficient estimators of the population parameters than the procedure based on simple random sampling (SRS), with the same sample size. In this paper, we compare the coverage probabilities of confidence intervals for the population standard deviation using the simple random sampling and the ranked set sampling. The following confidence intervals are considered: the exact, the Bonett, the Steve large sample normal approximations, the log asymptotic approximation and the adjusted degrees of freedom. The results for the Gamma, Log-normal and Exponential distributions and for the real data set are presented. The simulation study shows that the results obtained using the ranked set sampling are better than those using the simple random sampling.*

Keywords: *population standard deviation; confidence interval; coverage accuracy; ranked set sampling; simple random sampling*

1. INTRODUCTION

In this paper, we construct the confidence intervals for the population standard deviation. The population standard deviation is the most common scale parameter. The existing confidence interval for the estimation of the population standard deviation is the exact confidence interval, based on the statistic which has χ^2 distribution. This interval is appropriate if the distribution of the data is normal with no outliers. We are interested in confidence intervals which are appropriate if the data are not from the normal distribution, but from skewed distribution or have heavy tails. There are some alternatives to the exact confidence interval which can be used in such situations. There are no many authors who dealt with the confidence intervals that were less sensitive to the departure from normality and/or presence of outliers. Bonett (2006) proposed an approximate confidence interval for the population standard deviation which results were close to the exact confidence interval under the normality and had very good small-sample properties under the moderate non-normality. Cojbasic and Loncar (2011) and Cojbasic and Tomovic (2007) used the resampling methods for construction of confidence intervals for the population variance (taking the square root of the endpoints of that intervals gave the confidence intervals for the population standard deviation). Abu-Shawiesh et al. (2011) and Banik et al. (2014) conducted the large simulation studies in which compared the performances of the different confidence intervals for the standard deviation under the symmetric and skewed conditions. Hummel et al. (2005) proposed two alternative methods for finding the confidence interval for the standard deviation.

Ranked set sampling (or shortly RSS) is an alternative method of data collection and presents the cost-effective sampling procedure. The RSS is used for improving the estimators in the situations where the ranking of the units can be done easily compared to the effort required for the actual measurement of the variable of interest. This method was first proposed by McIntyre (1952). He estimated the mean of the population using the RSS instead of simple random sampling (or shortly SRS). Dell and Clutter (1972) showed that the ranked set sampling provided more precise estimates of the mean when the ranking of the units in the sample was easy. Using the RSS, Stokes (1980) concluded that an estimator of variance was asymptotically unbiased regardless of the errors that could occur during the ranking and that asymptotic efficiency of that estimator was better relative to the estimator based on the same number of the measured units from the random sample. Chen (2007) gave the review of the several variants of the ranked set sampling and presented some recent applications of that method. Samawi (1999) showed that the performance of the Monte Carlo methods, such as an importance or control variate sampling, were improved a lot using the ranked set sampling. Wolfe (2012) wrote the review article about the impact of the ranked set sampling on the statistical inference. Ganeslingam and Ganesh (2006) applied the ranked set sampling procedure on the estimation of the population mean and ratio using the real data set on the body measurements. Husby et al. (2005) used the crop production dataset from the United States Department of Agriculture to show the advantages of the RSS relative to the frequently used simple random sampling in the estimation of the mean and median of the population. Terpstra and Wang (2008) examined the several

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methods for construction of confidence intervals for the population proportion based on the RSS. Albatineh et al. (2014) performed the simulation study in which evaluated the performance of the several confidence intervals for the population coefficient of variation, using the coverage probabilities and the width of the intervals. Albatineh et al. (2017) constructed the confidence intervals for the Signal-to-Noise ratio using the RSS. More about the ranked set sampling methodology and its application can be found in Ozturk (2018), Zamanzade and Mahdizadeh (2017), Zamanzade and Vock (2015), Zhang et al. (2016), etc.

In this paper, we examine the confidence intervals for the population standard deviation which are more adequate to use when the data do not follow the normal distribution. We construct the confidence intervals using the simple random sampling and the ranked set sampling. The examined intervals are implemented using the R programming language. We generate random data from the Gamma, Log-normal and Exponential distributions, respectively and compare the coverage probabilities of the presented confidence intervals. Then, we apply the considered intervals to the measure of the systematic risk data.

The goal of this paper is to present the ranked set sampling procedure for construction of confidence intervals for the population standard deviation. The contribution of this paper is to emphasize the advantages of the ranked set sampling procedure over the simple random sampling procedure. The paper is organized as follows: in Section 2, we describe the ranked set sampling methodology; in Section 3, we present the confidence intervals for the population standard deviation; in Section 4, we conduct the simulation study for the data from the Gamma, Log-normal and Exponential distributions and for the real data set; in Section 5, we summarize results and draw the conclusions.

2. RANKED SET SAMPLING METHODOLOGY

The ranked set sampling procedures can be balanced or unbalanced. Each procedure can be with the perfect or imperfect ranking process. In this paper, we consider the balanced RSS with the perfect ranking process (see Ganeslingam and Ganesh (2006), Wolfe (2012), Albatineh et al. (2014)). The process of generating the balanced RSS involves drawing k^2 units at random from the population. After that, these units are randomly divided into *k* sets of *k* units each (we get *k* simple random samples of the size *k*). Within each set, the units are ranked according to the variable of interest. The perfect ranking process implies that actual measurements of the variable of interest on the selected units are done and that ranking is based on them. Opposite to the perfect ranking process, the imperfect ranking process includes visual comparisons of the units or the use of the auxiliary varables. After the ranking process, from the first set we select the unit with the smallest rank $X_{(1)}$ (if the ranking is perfect or $X_{[1]}$, if the ranking is imperfect). The remaining *k*-1 units are not considered further. Then, from the second set we select the unit with the second smallest rank $X_{(2)}$, and so on, until we select the unit with the largest rank from the *k*-th set, $X_{(k)}$. This procedure results in *k* observations $X_{(1)}, X_{(2)}, \ldots, X_{(k)}$ and is called the cycle. The number of the units in each simple random sample, *k*, is called the set size. If we want to obtain the balanced ranked set sample of size $n = mk$, we repeat the cycle *m* times (see Table 1). The complete balanced RSS with set size *k* and *m* cycles is given by $\left\{X_{(j)i} : j = 1, 2, \ldots, k; i = 1, 2, \ldots, m\right\}$. The term $X_{(j)i}$ is called the *j*-th order statistic from the *i*-th cycle.

Table 1: The balanced RSS with *m* cycles and set size *k*

Cycle 1	$X_{(1)1}$	$X_{(2)1}$	…	$X_{(k)1}$
Cycle 2	$X_{(1)2}$	$X_{(2)2}$	…	$X_{(k)2}$
…	…	…	…	…
Cycle *m*	$X_{(1)m}$	$X_{(2)m}$	…	$X_{(k)m}$

Source: Wolfe (2012)

The estimators of the mean and the variance of the population, based on the RSS, are given with the following formulas (see Stokes, 1980):

$$\bar{X}_{RSS} = \frac{1}{km} \sum_{j=1}^{k} \sum_{i=1}^{m} X_{(j)i}, \tag{1}$$

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$$S^2_{RSS} = \frac{1}{km-1}\sum_{j=1}^{k}\sum_{i=1}^{m}\left(X_{(j)i} - \bar{X}_{RSS}\right)^2. \tag{2}$$

3. CONFIDENCE INTERVALS FOR THE POPULATION STANDARD DEVIATION

In this section, we report five confidence intervals for the population standard deviation.

- The exact confidence interval

Let $X_1,...,X_n$ be independent and identically distributed random variables from the normal distribution, i.e. $X_i \sim N\left(\mu,\sigma^2\right)$. Let $S^2 = \left(1/(n-1)\right)\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$ be a sample variance. The statistic $(n-1)S^2/\sigma^2$ has χ^2 distribution with $n-1$ degrees of freedom. The exact $(1-\alpha)\cdot 100\%$ confidence interval for the population standard deviation, based on the previous statistic, is of the form:

$$\sqrt{\frac{(n-1)S^2}{\chi^2_{\alpha/2,(n-1)}}} \le \sigma \le \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\alpha/2,(n-1)}}}, \tag{3}$$

Where $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ are the $\alpha/2$ and $1-\alpha/2$ percentiles of the χ^2 distribution with $n-1$ degrees of freedom.

The exact confidence interval (3) is very sensitive to minor violations of the normality assumption. In the cases of violations of the normality assumption, there are the confidence intervals which present the alternatives to the exact confidence interval. In remain of the section, we consider that confidence intervals.

- The Bonett confidence interval

Let $X_1,...,X_n$ be continuous, independent and identically distributed random variables with $E\left(X_i\right)=\mu$, $\operatorname{var}\left(X_i\right)=\sigma^2$ and the finite fourth moment. Bonett (2006) proposed the following estimator of the kurtosis, γ_4, which is asymptotically eqvivalent to the Pearson's estimator:

$$\bar{\gamma}_4 = n\cdot\sum_{i=1}^{n}\left(X_i - m\right)^4 / \left(\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right)^2,$$

Where *m* is a trimmed mean with trim-proportion equal to $1/\left\{2\cdot(n-4)^{1/2}\right\}$. This estimator tends to have less negative bias and smaller coefficient of variability than Pearson's estimator in the symmetric and skewed leptokurtic distributions. The $(1-\alpha)\cdot 100\%$ confidence interval for the population standard deviation can be written in the following form (see Bonett, 2006):

$$\sqrt{\exp\left[\ln\left(cS^2\right) - Z_{1-\alpha/2}se\right]} \le \sigma \le \sqrt{\exp\left[\ln\left(cS^2\right) + Z_{1-\alpha/2}se\right]}, \tag{4}$$

Where $c = n/\left(n - Z_{1-\alpha/2}\right)$, S^2 is the sample variance, $Z_{1-\alpha/2}$ is the $1-\alpha/2$ percentile of the Z distribution and $se = c\cdot\left[\left\{\bar{\gamma}_4\cdot(n-3)/n\right\}/(n-1)\right]^{1/2}$.

- The Steve large sample normal approximations confidence interval

Steve proposed the following $(1-\alpha)\cdot 100\%$ confidence interval for the population standard deviation (see Banik et al., 2014):

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$$\sqrt{\frac{S^2}{1-Z_{\alpha/2}\sqrt{\frac{\hat{\gamma}-1}{n}}}} \le \sigma \le \sqrt{\frac{S^2}{1+Z_{\alpha/2}\sqrt{\frac{\hat{\gamma}-1}{n}}}}, \qquad (5)$$

where S^2 is the sample variance, $Z_{\alpha/2}$ is the $\alpha/2$ percentile of the standardized normal distribution and $\hat{\gamma} = n \cdot \sum_{i=1}^{n}\left(X_i - \bar{X}\right)^4 / \left(\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right)^2$ is the kurtosis estimator.

- The log asymptotic approximation confidence interval (LOG CI)

The distribution of the sample variance, S^2, has the high skewness for small *n*. In order to reduce the skewness, Hummel et al. (2005) applied natural log to the sample variance in (5) and proposed the $(1-\alpha)\cdot 100\%$ confidence interval for the population standard deviation:

$$\sqrt{\left[S^2 \exp\left(Z_{\alpha/2}\sqrt{\frac{\hat{\gamma}-1}{n}}\right)\right]} \le \sigma \le \sqrt{\left[S^2 \exp\left(-Z_{\alpha/2}\sqrt{\frac{\hat{\gamma}-1}{n}}\right)\right]}, \qquad (6)$$

Where $Z_{\alpha/2}$ is the $\alpha/2$ percentile of the Z distribution and $\hat{\gamma}$ is the kurtosis estimator.

- The adjusted degrees of freedom confidence interval (ADF CI)

Hummel et al. (2005) adjusted the degrees of freedom of the exact confidence interval (3) and proposed the following $(1-\alpha)\cdot 100\%$ confidence interval for the population standard deviation:

$$\sqrt{\frac{\hat{r}S^2}{\chi^2_{\alpha/2,\hat{r}}}} \le \sigma \le \sqrt{\frac{\hat{r}S^2}{\chi^2_{1-\alpha/2,\hat{r}}}}, \qquad (7)$$

Where $\hat{r} = \dfrac{2n}{\hat{\gamma}_e + \left(\dfrac{2n}{n-1}\right)}$ and $\hat{\gamma}_e$ is the estimate of the kurtosis excess, which is defined as $\hat{\gamma}_e = \dfrac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{n}\dfrac{\left(X_i-\bar{X}\right)^4}{S^4} - \dfrac{3(n-1)^2}{(n-2)(n-3)}$. If the random sample is generated from the normal distribution, then $r = n-1$ and (7) reduces to (3).

The performance of all presented intervals will be considered using the SRS and the RSS. In order to get the estimators of the mean and variance of the population we will use the regular formulas for the SRS and Equations (1) and (2) for the RSS.

4. CASE STUDY

In this part of the paper, we present the results of applying the proposed methods on the simulated data and real-economic data.

4.1 Simulation study

In this part of our work we examine the coverage accuracy of two-sided confidence intervals for the standard deviation introduced in the Section 3. Our objective is to compare the performance of the confidence intervals for estimating the standard deviation using the SRS and the RSS. The nominal confidence level is set to 95% and we want to determine which of the proposed intervals will give the coverage probability that is the closest to 95%. For that purpose, we consider three scenarios. In the first scenario we deal with the Gamma distribution, while in the second scenario the subject of the consideration is the Log-normal distribution. In the third scenario we investigate the Exponential distribution. It is important to emphasize that we can deal with any other scenario with the other skewed distribution.

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In the first scenario, we consider the Gamma distribution with the shape parameter 2 and with the scaling parameters 0.5, 1.6 and 3.2. For each combination of the parameter setting and the sample size (15, 20, 50, 80), we performed 1000 simulations. All examined intervals are implemented using the *R* programming language. In Table 2 we present the results of the coverage accuracy of 95% confidence intervals for the standard deviation of the Gamma distribution. It can be seen that for the small samples (size 15), the RSS Bonett interval gives the best results, i.e. the coverage probabilities that are the closest to 0.95 (the coverage greater than 0.932). For the moderate samples (size 20), the RSS Bonett interval gives the best coverage (below 0.961). In the case of big samples (size 50), depending on the scaling parameter of the Gamma distribution, the best coverage probabilities are obtained using the Bonett and the RSS Log intervals (the coverage greater than 0.939). For the samples of size 80, the Bonett interval and the RSS ADF interval give the best coverage accuracy (above 0.943).

Table 2: The coverage of 95% two-sided confidence intervals for the standard deviation of the Gamma distribution

a	***s***	***n***	χ^2	**RSS χ^2**	**Bonett**	**RSS Bonett**	**Steve**	**RSS Steve**	**Log**	**RSS Log**	**ADF**	**RSS ADF**
2	0.5	15	0.831	0.865	0.937	0.939	0.714	0.748	0.797	0.821	0.841	0.895
		20	0.838	0.881	0.909	0.960	0.769	0.773	0.788	0.864	0.857	0.907
		50	0.784	0.818	0.957	0.966	0.902	0.932	0.903	0.923	0.907	0.935
		80	0.803	0.861	0.945	0.958	0.913	0.936	0.923	0.943	0.926	0.952
2	1.6	15	0.824	0.887	0.911	0.936	0.750	0.757	0.767	0.784	0.835	0.883
		20	0.854	0.875	0.924	0.961	0.726	0.747	0.795	0.830	0.829	0.911
		50	0.805	0.898	0.933	0.972	0.900	0.923	0.885	0.939	0.908	0.934
		80	0.847	0.852	0.958	0.976	0.918	0.939	0.915	0.932	0.921	0.943
2	3.2	15	0.829	0.871	0.910	0.932	0.723	0.737	0.797	0.809	0.883	0.893
		20	0.807	0.868	0.923	0.952	0.750	0.832	0.840	0.845	0.853	0.905
		50	0.765	0.856	0.936	0.982	0.901	0.923	0.885	0.939	0.919	0.932
		80	0.758	0.840	0.952	0.965	0.907	0.941	0.905	0.938	0.922	0.936

In the second scenario, we deal with the Log-normal distribution with the shape parameter 2 and with the scaling parameters 0.25, 0.5 and 0.6. For each combination of the parameter setting and the sample size (15, 20, 50, 80), we generated 1000 samples. All considered intervals are implemented using the *R* programming language. In Table 3 we present the results of the coverage accuracy of 95% intervals for the standard deviation of the Log-normal distribution. For the small samples, the RSS χ^2 and the RSS Bonett intervals give the coverage probabilities that are the closest to 0.95 (above 0.924). In the case of the moderate samples, depending on the scaling parameter of the Log-normal distribution, the RSS χ^2 interval and the RSS Bonett interval give the best results (the coverage greater than 0.933). For the big samples (size 50) the best results are obtained with the Steve interval and the RSS Bonett interval (above 0.940). For the samples of size 80, the Steve, the RSS ADF, the Bonett and the RSS Bonett intervals give the coverage probabilities that are the closest to 0.95.

In the third scenario, we investigate the Exponential distribution with the rate parameters 0.5, 1.5 and 2.2. For each parameter setting and each sample size (15, 20, 50, 80), we performed 1000 simulations. All considered intervals are implemented using the *R* programming language. In Table 4 we present the results of the coverage accuracy of 95% confidence intervals for the standard deviation of the Exponential distribution. It can be seen that for the small samples, the RSS Bonett interval gives the best results (above 0.923). For the moderate samples, the RSS Bonett interval gives the best coverage (greater than 0.938). In the case of big samples (size 50) the best coverage probabilities are obtained using the RSS Bonett interval (above 0.945). For the samples of size 80, depending on the rate parameter of the Exponential distribution, the Bonett and the RSS Bonett intervals give the best coverage accuracy (above 0.942).

Table 3: The coverage of 95% two-sided confidence intervals for the standard deviation of the Log-normal distribution

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μ	σ	**n**	χ^2	**RSS** χ^2	**Bonett**	**RSS Bonett**	**Steve**	**RSS Steve**	**Log**	**RSS Log**	**ADF**	**RSS ADF**
2	0.25	15	0.911	0.946	0.965	0.977	0.788	0.839	0.807	0.852	0.904	0.924
		20	0.920	0.937	0.964	0.980	0.833	0.883	0.864	0.874	0.915	0.935
		50	0.909	0.918	0.972	0.974	0.947	0.954	0.923	0.936	0.927	0.963
		80	0.912	0.938	0.969	0.980	0.945	0.964	0.929	0.943	0.942	0.955
2	0.5	15	0.817	0.846	0.922	0.942	0.658	0.690	0.747	0.759	0.769	0.838
		20	0.758	0.850	0.930	0.949	0.679	0.729	0.768	0.818	0.800	0.850
		50	0.669	0.828	0.937	0.953	0.911	0.927	0.872	0.910	0.904	0.914
		80	0.762	0.766	0.949	0.961	0.920	0.936	0.905	0.932	0.920	0.925
2	0.6	15	0.734	0.753	0.915	0.924	0.610	0.636	0.676	0.753	0.777	0.802
		20	0.709	0.737	0.900	0.933	0.579	0.612	0.749	0.760	0.785	0.840
		50	0.655	0.671	0.919	0.940	0.842	0.884	0.791	0.884	0.867	0.875
		80	0.556	0.682	0.939	0.941	0.866	0.888	0.862	0.886	0.884	0.897

Table 4: The coverage of 95% two-sided confidence intervals for the standard deviation of the Exponential distribution

λ	**n**	χ^2	**RSS** χ^2	**Bonett**	**RSS Bonett**	**Steve**	**RSS Steve**	**Log**	**RSS Log**	**ADF**	**RSS ADF**
0.5	15	0.757	0.799	0.904	0.930	0.614	0.637	0.763	0.769	0.781	0.797
	20	0.753	0.843	0.911	0.943	0.586	0.722	0.753	0.832	0.844	0.858
	50	0.720	0.762	0.938	0.958	0.813	0.907	0.848	0.891	0.904	0.910
	80	0.722	0.751	0.943	0.960	0.908	0.930	0.903	0.917	0.907	0.925
1.5	15	0.753	0.763	0.905	0.935	0.510	0.654	0.746	0.795	0.764	0.807
	20	0.769	0.786	0.931	0.953	0.631	0.763	0.770	0.836	0.797	0.836
	50	0.683	0.751	0.918	0.960	0.822	0.901	0.888	0.902	0.900	0.925
	80	0.658	0.744	0.942	0.963	0.916	0.934	0.914	0.920	0.918	0.933
2.2	15	0.746	0.755	0.904	0.923	0.625	0.652	0.720	0.726	0.780	0.839
	20	0.694	0.785	0.914	0.938	0.647	0.721	0.704	0.808	0.778	0.840
	50	0.687	0.734	0.931	0.945	0.883	0.897	0.875	0.908	0.893	0.906
	80	0.653	0.739	0.933	0.966	0.908	0.922	0.894	0.917	0.906	0.922

4.2. Application to the real data

In this part of the paper, we analyze the measure of the systematic risk data in 490 companies (it is about S&P500, but there are no data for some companies) on the 5th July 2017. The data are from the website http://finance.yahoo.com/.

In analysis of securities, the measure of the systematic risk (beta) takes a central place. The measure of the systematic risk represents the measure of the sensitivity of the yield of the securities to the changes in the yield on the market. Beta shows that if the yield on the market changes by one percent, by how many percentage points the yield of the securities will change.

Descriptive statistics for the analyzed data are given in Table 5. Figures 1 represents the histogram of the analyzed variable. We can see that the measure of the systematic risk is not normally distributed. Also, we used the Shapiro-Wilk normality test to examine whether the beta was normally distributed. The test showed the same result as the histogram (p-value is approximately 0).

Table 5: Descriptive statistics for the data set

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Variable	*N*	Mean	Std. deviation	Skewness coef.
The measure of the systematic risk	490	1.02	0.52	0.43

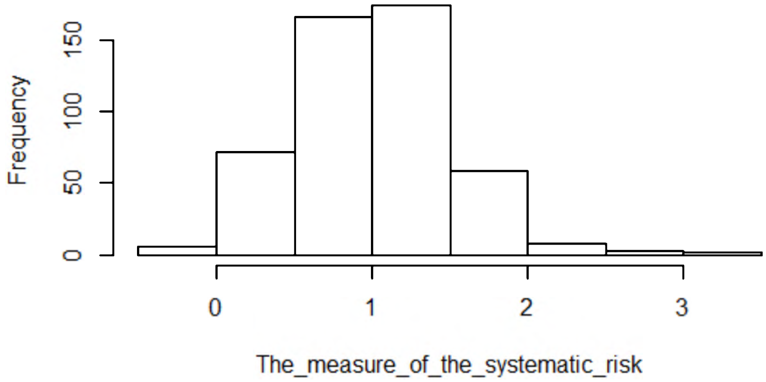

Figure 1: Histogram of the measure of the systematic risk

Results of the coverage accuracy of 95% confidence intervals for the standard deviation of the data set are given in Table 6. It can be seen that for the small samples, the RSS χ^2 interval gives the best coverage (0.945). For the moderate samples, the RSS ADF interval gives the coverage that is the closest to 0.95. For the samples of size 50, the RSS Steve interval is the best choice and in the case of the samples of size 80, the ADF interval gives the best coverage accuracy (0.946).

Table 6: The coverage of 95% two-sided confidence intervals for the standard deviation: the data set

n	χ^2	RSS χ^2	Bonett	RSS Bonett	Steve	RSS Steve	Log	RSS Log	ADF	RSS ADF
15	0.934	0.945	0.962	0.975	0.811	0.833	0.839	0.879	0.907	0.919
20	0.918	0.924	0.972	0.977	0.878	0.893	0.873	0.898	0.926	0.935
50	0.921	0.943	0.976	0.986	0.919	0.955	0.904	0.920	0.921	0.943
80	0.911	0.921	0.971	0.968	0.938	0.931	0.913	0.915	0.946	0.938

5. CONCLUSIONS

In this paper, we used the simple random sampling and the ranked set sampling to compare the coverage probabilities of confidence intervals for the population standard deviation. The exact confidence interval, the Bonett, the Steve large sample normal approximations, the log asymptotic approximation and the adjusted degrees of freedom confidence intervals were examined.

In the first scenario, we investigated the Gamma distribution. For the small and moderate samples, the RSS Bonett interval gave the coverage probabilities that were the closest to 0.95, whereas for the big samples the Bonett, the RSS Log and the RSS ADF intervals gave the best results. In the second scenario, we dealt with the Log-normal distribution. For the small and moderate samples, the best results were obtained using the RSS χ^2 and the RSS Bonett intervals. In most cases, for the big samples, the Steve interval and the RSS Bonett interval gave the best coverage. In the third scenario, we considered the Exponential distribution. For the small and moderate samples, the RSS Bonett interval was the best choice, whereas for the big samples, the Bonett and the RSS Bonett intervals gave the best results.

The analysis of the real data showed that for the small samples, the RSS χ^2 interval gave the best coverage accuracy, while for the moderate samples the best choice was the RSS ADF interval. For the big samples, the RSS Steve and the ADF interval gave the best results. We can see that using the RSS gives much better coverage probabilities, so we recommend using it when construct the confidence intervals for the population standard deviation.

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