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Measuring Downside Risk in Portfolios with Bitcoin

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Abstract

This study aims to determine which auxiliary asset – S&P500, SHCOMP, the U.S. 10Y bond, gold, Brent or corn, in combination with Bitcoin has the best downside risk-minimizing performances. Six portfolios are constructed via an optimal DCC-GARCH model, while for downside risk measures, we use parametric and semiparametric Value-at-Risk and Conditional Value-at-Risk. All selected auxiliary assets have very low dynamic correlation with Bitcoin, which classifies them as good diversifiers. According to parametric results, S&P500 has the best downside risk-minimizing output, while SHCOMP and gold take second and third place. However, when higher moments of portfolios are taken into account, the results change significantly. Due to very high kurtosis and negative skewness, portfolio with S&P500 has among the worst semiparametric downside risk results. On the other hand, SHCOMP index and gold have relatively favourable third and fourth moments' characteristics, which pushes them to the first and second place of the best auxiliary assets when modified downside risk measures are at stake. We also calculate Sharpe ratio, which suggests that portfolio with gold has by far the best return/risk characteristics.

1. Introduction

In 2008, a new type of global asset was born, cryptocurrency named Bitcoin, which decentralized transaction system is based on blockchain technology. Since the appearance of Bitcoin in 2009, well over 100 different crypto currencies are announced, but Bitcoin remains the most significant and well-known cryptocurrency, capturing more than 50% of coin market capitalization (Al-Yahyaee et al., 2018; Mensi et al. 2019). This instrument, which some rather prefer to call an investment vehicle, was introduced by a person or a group of people under the pseudonym Satoshi Nakamoto (see Kliber and Wlosik, 2019). Unlike regular currencies, which are issued and guaranteed by the state, cryptocurrencies are based on cryptography and they do not have classical functions of money – a medium of exchange for everyday purposes, store of value and unit of account. According to Baur et al. (2018), the attractiveness of Bitcoin stems from the fact that cryptocurrencies has low transaction costs, comparing to mainstream currencies, a government-free design, a

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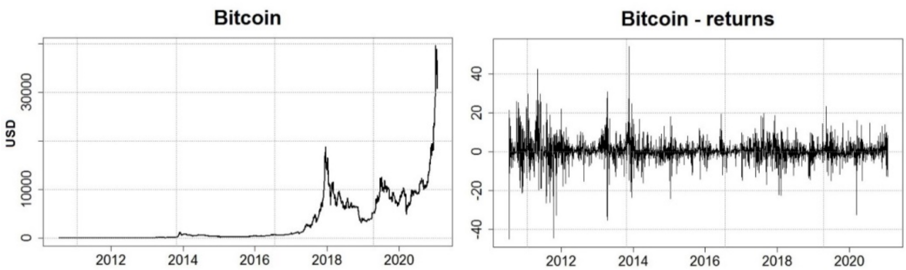
peer-to-peer system and the possibility for instant payments to any party without any involvement of financial intermediaries. Many market participants consider Bitcoin as very appealing investment tool, because Bitcoin price has a tendency to rise. For instance, Bitcoin skyrocketed from less than 10 USD in 2011 to more than 30.000 USD at the beginning of 2021 (see left plot in Figure 1). In addition, Bitcoin has favourable diversification properties, which is mirrored in a form of negative or low correlation with other asset classes, such as stocks, bonds and various commodities. On the other hand, Bitcoin also has its drawbacks. For instance, some investors see Bitcoin as a speculative bubble or Ponzi scheme as it has very weak or even no fundamental value, as Uddin et al. (2020) claimed. Also, very distinctive feature of Bitcoin is its highly volatile nature, which is a direct consequence of speculations (see right plot in Figure 1). Let and Siemaszkiewicz (2020) asserted that Bitcoin price can rise and fall dramatically several times for a single day, which suggests that investing in Bitcoin is subject to a large risk and volatility. Therefore, due to unpredictable and erratic behaviour of Bitcoin price, it would be useful for investors to know with which auxiliary asset to combine Bitcoin in a two-asset portfolio, in order to mitigate risk of such portfolio.

Figure 1 Empirical Dynamics of Bitcoin Price and its Returns



According to the aforementioned, this paper combines six heterogeneous assets with Bitcoin with goal to find out which portfolio has the best risk-minimizing properties. The auxiliary assets are: the U.S. 10Y bonds, gold futures, Brent oil futures, S&P500 index, SHCOMP index and corn futures. The reason why we consider relatively wide range of different assets is the fact that all these investment instruments behave intrinsically different, because they are all driven by different fundamental factors that eventually affect their own supply and demand structure. In particular, bonds are subject to the changes in interest rate, level of inflation and monetary policy. Stock indices are affected by economic growth, market sentiment, expectations, relation with other markets. This is the reason way we choose one composite stock index of developed country (S&P500), and another one of fast-growing emerging country, such as China (SHCOMP). Besides, we also opt for three different types of commodities (gold, Brent oil and corn), and each commodity has its distinctive factors that determine their prices, such as weather, geopolitical events, global supply and demand conditions. We consider futures prices rather than spot prices of the commodities, because, futures prices, by definition, more realistically reflect all information and all events that occur globally. More specifically, futures markets process global information faster than spot markets and embed new information in prices more effectively.

In the process of portfolio designing, we calculate dynamic optimal in-sample weights of auxiliary assets *via* Kroner and Ng (1998) equation. This equation produces minimum-variance portfolio by default. In order to be as much as accurate in this procedure, we obtain inputs for this equation *via* several bivariate dynamic conditional correlation GARCH (DCC-GARCH) models, also considering three different multivariate distributions. In particular, for every pair of assets that is selected, we use four different GARCH specifications in DCC framework, i.e. symmetric GARCH, two traditional asymmetric models – GJRGARCH and EGARCH, and one untraditional asymmetric model – NAGARCH of Engle and Ng (1993). Three different multivariate distributions (MVD) are used in this process – normal, Student t and Laplace, because we assume that an optimal MVD can produce more accurate dynamic correlation, which can be reflected directly on more precise weights of an auxiliary asset. The best multivariate model is chosen based on the lowest Akaike information criterion (AIC). Numerous previous papers utilized multivariate DCC model to construct a minimum variance portfolio (see e.g. Yousaf and Ali, 2020; Abdulkarim et al., 2020; Živkov et al., 2021)

After the construction of six portfolios, we try to measure their performance in terms of the lowest risk. Minimum variance portfolio of Kroner and Ng (1998) incorporates both upside and downside risk, assigning an equal weight to positive and negative returns. However, investors usually rather prefer to know the downside risk of the hedged portfolio, according to Altun et al. (2017), Altun et al. (2017), Grané and Veiga (2012). Therefore, we observe portfolio risk from the aspect of downside risk, and in that respect compute four risk metrices. First, we calculate parametric Value-at-Risk (VaR), which assumes that portfolio follows normal distribution. Value-at-Risk measures the maximum loss that an investment portfolio is likely to face, taking into account a specified time-frame with a certain probability level. Besides VaR metrics, we also calculate parametric conditional Value-at-Risk (CVaR), which overcomes shortages of VaR and measures the mean loss, conditional upon the fact that VaR has been exceeded. In addition, it is well known that investors require a higher risk premium for portfolios that exhibit negative skewness and higher kurtosis. This means that investors have to account third and fourth moments of portfolio return distribution in addition to volatility when measure the risk. In terms of our research, this fact adds up very well with the assertions of Sheraz and Dedu (2020) and Koo et al. (in press), who stated that cryptocurrencies generally have a tendency towards more massive negative intra-day tails. In this regard, we calculate two additional semiparametric risk measures, based on a Cornish-Fisher expansion (Cornish and Fisher, 1938), and these are modified VaR and modified CVAR. Modified VaR was introduced by Favre and Galeano (2002).

It should be underlined that traditional VaR method is not an efficient measure of risk, if returns of a portfolio do not follow Gaussian distribution. In that respect, modified VaR (mVAR) could be very handy tool in overcoming this issue, because it takes into account the higher moments – skewness and kurtosis. Skewness tells about the tilt of distribution, while kurtosis indicates the fat-tails. Distributions with negative skewness and fat-tails is called platykurtic, and if this is the case, modified VaR will penalize these unfavourable characteristics of distribution, and consequently give a larger estimation for the loss in comparison with the conventional VaR. On the other hand, when distribution has positive skewness and lower kurtosis, then mVAR rewards these distribution features, meaning that calculated mVaR loss will be smaller than traditional VaR. When skewness and kurtosis have Gaussian properties, then this metrics converges to the usual parametric VaR. In order to be consistent with the previously explained risk measures, besides mVaR, we also calculate modified conditional VaR (mCVaR), which gives us an opportunity to compare the downside risk measures calculated in both traditional (VaR and CVaR) and more elaborate way (mVaR and mCVaR).

To the best of our knowledge, this is the first paper that uses relatively wide range of different auxiliary assets for the diversification purposes with Bitcoin, applying at the same time sophisticated approaches for portfolio construction and downside risk measurement.

Besides introduction, the rest of the paper is structured as follows. Second section presents literature review. Third section explains used methodologies. Fourth section gives a dataset overview. Fifth section is divided in three subsections, presenting DCC estimates and the results of parametric and semiparametric VaR and CVaR. Sixth section concludes.

2. Literature Review

As a new asset class, cryptocurrencies gained considerable attention from researchers in the past decade. Thus, a number of studies exist, *inter alia*, in the field of international portfolio diversification. However, vast majority of the papers observe Bitcoin as a diversifier, whereas our study differentiates in a sense that we observe Bitcoin as an asset that needs to be diversified. Some of the existing paper in this area are presented in the following. For instance, Dyhrberg (2016) found hedging power of Bitcoin, because it has week correlation with major equities, oil and currencies. Using GARCH models for the research, he reported that Bitcoin may be useful in risk management and ideal for risk-averse investors, since Bitcoin can successfully anticipate negative shocks to the market. Symitsi and Chalvatzis (2019) presented an analysis of benchmark portfolios of currencies, gold, oil and stocks as well as a multi-asset portfolio of currencies, gold, oil, stock, real estate and bond with respective portfolios that invest additionally in Bitcoin under four trading strategies. They determined economic gains for portfolios when Bitcoin is included, and this applies for both bullish and bearish cryptocurrency market conditions. They reported significant diversification benefits for equal-weighted and optimal minimum-variance portfolios with daily and weekly rebalancing. Their results suggested that Bitcoin portfolios come with high Sharpe ratios, and this remained even in cases when the participation of Bitcoin is very small, such as in minimum-variance portfolios. Bouri et al. (2017) researched the relationship between Bitcoin and energy commodities. Based on the results, he claimed that Bitcoin is a strong hedge and a safe-haven against movements in commodities. in addition, they also asserted that Bitcoin enjoys hedging and safe-haven properties before December 2013 oil price crash, whereas no such advantage exists in the post-crash period. The study of Bouri et al. (2020) compared the safe-haven properties of Bitcoin, gold, and the commodity index against world, developed, emerging, USA, and Chinese stock market indices.

They used the wavelet coherency approach and show that the overall dependence between Bitcoin/gold/commodities and the stock markets is not very strong at various time scales, whereas Bitcoin is the least dependent. They also researched the diversification potential at the tail of the return distribution via wavelet Value-at-Risk approach and reveal that the degree of co-movement between gold and stock returns affects the portfolio's VaR level. They underlined that the benefits of diversification vary in the time-frequency space, while Bitcoin has a superiority over both gold and commodities.

The paper of Rehman et al. (2020) investigated the risk dependence between daily Bitcoin and major Islamic equity markets. They reported that Islamic indices – DJIUK, DJIJP and DJICA exhibit time varying dependence with Bitcoin. They concluded that Islamic equity market serves as an effective hedge in a portfolio along with Bitcoin. Matkovskyy et al. (2021) analysed the ability of the top 10 cryptocurrencies in enhancing portfolio returns of the 10 worst-performing stocks in the S&P600, S&P400 and S&P100 indexes. They used probabilistic utility approach with different algorithms and time horizons and reported that adding cryptocurrencies to traditional stock portfolios increases value in terms of enhancing returns. Demiralay and Bayraci (in press) studied the time-varying investment benefits of cryptocurrencies for stock portfolios using a correlation-based conditional diversification benefits (CDB) measure. They designed six portfolios consisting of cryptocurrencies, developed and emerging equity markets. Their results indicated that the time-varying correlations between cryptocurrencies and stock markets are generally low, but the level of correlations significantly increases in turbulent periods. They concluded that adding cryptocurrencies to equity market portfolios enhances portfolio diversification. Pho et al. (2021) compared Bitcoin with gold in the diversification of Chinese portfolios. They used a new copula-based joint distribution function of returns to simulate the Value-at-Risk and CVaR of portfolios including Bitcoin (or gold) and those without it. Their results showed that gold is a better portfolio diversifier than Bitcoin, because it helps better reduce the risk of portfolios. On the other hand, Bitcoin better increases the return, but it also increases the risk.

3. Methodologies

3.1 Dynamic Conditional Correlation Model

In order to construct minimum-variance portfolio, we have to calculate conditional volatilities of two instrument and their respective conditional covariance. In that effort, we use an optimal bivariate DCC model, which means that we estimate four different univariate GARCH specification under three different multivariate distributions for every estimated pair of assets. Since daily financial time-series usually reports clustering phenomenon and leverage effect in the volatility, we try to recognize these stylized facts in the best possible way, so we consider four GARCH specifications – simple GARCH, GJRGARCH, EGARCH and NAGARCH models in DCC framework. The lowest Akaike information criterion (AIC) indicates the optimal GARCH model. The mean equation of all univariate GARCH models has a form of AR(1), which is enough lag-order to handle serial correlation problem in the selected time-series. Mathematical formulation of the mean equation and four different aforementioned GARCH specifications are presented in equations (1) to (5), respectively.

$$y_t = C + \phi y_{t-1} + \varepsilon_t; \qquad \varepsilon_t \sim z_t \sqrt{\sigma_t^2} \tag{1}$$

$$\sigma_t^2 = c + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

$$\sigma_t^2 = c + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1}; \qquad I_{t-1} = \begin{cases} 1 \ if \ \ \varepsilon_{t-1} < 0 \\ 0 \ if \ \ \varepsilon_{t-1} > 0 \end{cases} \tag{3}$$

$$\ln(\sigma_t^2) = C + \alpha \left| \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \right| + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}, \tag{4}$$

$$\sigma_t^2 = c + \beta \sigma_{t-1}^2 + \alpha \sigma_{t-1}^2 (\varepsilon_{t-1}^2 - \gamma)^2 \tag{5}$$

where C and c are constants in the mean and variance equations, while ϕ is an autoregressive parameter. y_t stands for 2×1 vector of Bitcoin and particular asset returns, $y_t = [y_t^{Bitcoin}, y_t^{asset}]'$, whereas ε_t is 2×1 vector of error terms $\varepsilon_t = [\varepsilon_t^{Bitcoin}, \varepsilon_t^{asset}]'$. Symbol z_t describes an independently and identically distributed process, which has a form of a Student-t distribution: $z_t \sim t(0,1)$. Regarding conditional variance equations, parameter β captures the persistence of volatility, while α measures an ARCH effect. Parameter γ gauges presence of an asymmetric effect. If $\gamma > 0$ than negative shocks impact volatility more than positive shocks, and *vice-versa*.

As for multivariate model, we use specification of Engle (2002), which computes conditional correlation in two-step procedure. By first, the selected time-series are fitted in a specific univariate GARCH model in order to obtain conditional standard deviation, $\sqrt{\sigma_t^2}$. Afterwards, asset-return residuals are standardized, i.e. $\nu_t = \varepsilon_t / \sqrt{\sigma_t^2}$, whereby ν_t is used subsequently to estimate the parameters of the conditional correlation. According to Engle (2002), the multivariate conditional variance has a form as: $\Sigma_t = D_t C_t D_t$, where $D_t = diag(\sqrt{\sigma_{11,t}^2} \cdots \sqrt{\sigma_{nn,t}^2})$ and $\sigma_{ii,t}^2$ represents the conditional variance obtained from some form of univariate GARCH model in the first step. The evolution of correlation in the DCC model is given as in equation (6):

$$Q_t = (1 - a - b)\bar{Q} + \alpha \nu_{t-1} \nu_{t-1}^{'} + \beta Q_{t-1}, \tag{6}$$

where a and b depict nonnegative scalar parameters under condition $a + b < 1$; $Q_t = (q_{ij,t})$ is n × n time-varying covariance matrix of residuals, while $\bar{Q} = E[\nu_t \nu_t']$ stands for n × n time-invariant variance matrix of ν_t. Since Q_t does not have unit elements on the diagonal, it is scaled to obtain proper correlation matrix (C_t) according to following form: $C_t = (diag(Q_t))^{-1/2} Q_t (diag(Q_t))^{-1/2}$. Accordingly, the element of C_t looks like:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} = \frac{(1-a-b)\bar{q}_{ij} + a\nu_{i,t-1}\nu_{j,t-1} + bq_{ij,t-1}}{\sqrt{\left[(1-a-b)\bar{q}_{ii} + a\nu_{i,t-1}^2 + bq_{ii,t-1}\right]}\sqrt{\left[(1-a-b)\bar{q}_{jj} + a\nu_{j,t-1}^2 + bq_{jj,t-1}\right]}} \quad (7)$$

where i ≠ j, and in our bivariate model, n is equal to 2. All DCC models were estimated by a quasi-maximum likelihood (QMLE) technique. As we have said earlier, all DCC models are estimated using three different multivariate distribution functions – normal, Student t and Laplace. Normal and Student t are well known distributions, so we present only mathematical function of less known Laplace distribution in equation (8), in order to be parsimonious as much as possible.

$$f(r_t; m, \Omega_t) = \frac{2\exp(r_t'\Omega_t^{-1}m)}{(2\pi)^{N/2}|\Omega_t|^{1/2}} \left(\frac{r_t'\Omega_t^{-1}r_t}{2 + m'\Omega_t^{-1}m}\right)^{\nu/2} K_\nu\left(\sqrt{(2 + m'\Omega_t^{-1}m)(r_t'\Omega_t^{-1}r_t)}\right) \quad (8)$$

3.2 Minimum-Variance Portfolio Construction

After estimation of an optimal DCC model and obtained conditional variances and covariance for every pair of assets, we are able to construct a two-asset portfolio *via* Kroner and Ng (1998) equation. Portfolio constructed in this way minimizes unsystematic risk without affecting the potential of expected returns. Kroner and Ng (1998) equation calculates optimal dynamic portfolio weight of an auxiliary asset (W_t^{asset}), and its form is presented in equation (9), while the equation restrictions are given in equation (10).

$$W_t^{asset} = \frac{\sigma_t^{2\,(Bitcoin)} - COV_t^{(Bitcoin,asset)}}{\sigma_t^{2\,(Bitcoin)} - 2COV_t^{(Bitcoin,asset)} + \sigma_t^{2\,(asset)}} \quad (9)$$

$$W_t^{asset} = \begin{cases} 0, & if \quad W_t^{asset} < 0 \\ W_t^{asset}, & if \quad 0 < W_t^{asset} < 1 \\ 1, & if \quad W_t^{asset} > 1 \end{cases} \quad (10)$$

where W_t^{asset} denotes the weight of a particular asset in 1\$ portfolio of two-asset holding at time t. Symbols $\sigma_t^{2\,(Bitcoin)}$ and $\sigma_t^{2(asset)}$ indicate to conditional variances of Bitcoin and selected assets, respectively. $COV_t^{(Bitcoin,asset)}$ refers to conditional covariance between Bitcoin and some asset, at time t. The weight of Bitcoin in two-asset portfolio is calculated as $1 - W_t^{asset}$.

3.3 Downside Risk Measurements

Kroner and Ng (1998) equation produces a portfolio that minimizes variance, incorporating both downside and upside risk. However, most investors are keen to know only the size of downside risk (see e.g. Miletić and Miletić, 2015; Hong, 2017). Therefore, we compute four different downside risk measures. We start with parametric VaR, which measures a loss that investor might endure in a single day under certain probability. However, Value-at-Risk has a shortcoming being that it disregards any loss beyond the VaR level. In this respect, we also calculate CVaR metrics, which can indicate an average expected loss of a portfolio, and it is regarded as better indicator than VaR. However, both measures assume normal distribution of portfolio, which is usually inaccurate conjecture, particularly when we deal with financial data. In order to overcome possible biased measures of downside risk, we additionally calculate modified VaR and CVaR that are based on a Cornish–Fisher expansion approximation. These values take into account higher moments, i.e. skewness and kurtosis of the portfolio distribution and give more realistic estimate about downside risk that investor might endure. Parametric VaR for short position is calculated as in equation (11):

$$VaR_\alpha = \hat{\mu} + Z_\alpha \hat{\sigma} \tag{11}$$

where $\hat{\mu}$ and $\hat{\sigma}$ refer to the estimated mean and standard deviation of a particular portfolio, respectively, and Z_α stands for left quantile of the normal standard distribution.

We also consider Conditional Value-at-Risk that is introduced by Rockafellar and Uryasev (2000). CVaR measures the mean loss, conditional upon the fact that the VaR has been exceeded and it is calculated as in equation (12):

$$CVaR_\alpha = -\frac{1}{\alpha}\int_0^\alpha VaR(x)dx, \tag{12}$$

where $VaR(x)$ is Value-at-Risk of a particular two-asset portfolio. α denotes the left quantile of the standard normal distribution.

Assuming that portfolios with Bitcoin are probably polluted with heavy tails, we consider additional analytical expression for VaR and CVAR, which are based on the Cornish–Fisher expansion. Accordingly, mVaR for short position is defined as in the expression (13):

$$mVaR_\alpha = \hat{\mu} + Z_{CF,\alpha}\hat{\sigma}, \tag{13}$$

where $Z_{CF,\alpha}$ is the non-normal-distribution percentile adjusted for skewness and kurtosis according to the Cornish–Fisher Expansion:

$$Z_{CF,\alpha} = Z_\alpha + \frac{1}{6}(Z_\alpha^2 - 1)S + \frac{1}{24}(Z_\alpha^3 - 3Z_\alpha)K - \frac{1}{36}(2Z_\alpha^3 - 5Z_\alpha)S^2 \qquad (14)$$

where S and K are measures of skewness and kurtosis of a portfolio.

Accordingly, mCVaR specification is given in equation (15):

$$mCVaR_\alpha = -\frac{1}{\alpha}\int_0^\alpha mVaR(x)dx \qquad (15)$$

The evaluation of downside risk-reduction performances of the portfolios is achieved by Hedge effectiveness indices (HEI) for all aforementioned risk measures. In particular, portfolio HEI risk measure (HEI_{RM}) is calculated in the following way:

$$HEI_{RM} = \frac{RM_{unhedged} - RM_{hedged}}{RM_{unhedged}} \qquad (16)$$

where RM denotes particular down-side risk measure of a portfolio, i.e. VaR, CVaR, mVaR or mCVaR. Subscript $unhedged$ refers to investment only in Bitcoin, whereas the label $hedged$ indicates to investment in a two-asset portfolio with Bitcoin as primary investment. As much as HEI index is closer to 1, the higher hedging effectiveness is, and *vice-versa*.

4. Dataset

This paper considers daily data of Bitcoin and six other heterogeneous assets – the U.S. 10Y bonds, S&P500 and SHCOMP indices, and futures of gold, Brent oil and corn. All assets are denominated in USD, except SHCOMP index, which is in renminbi. It should be said that all portfolios with Bitcoin can be created at any time, because cryptocurrency markets work around the clock – 24/7.[1] The data span ranges from July 2010 to January 2021, and all time-series are collected from *Stooq.com* website. Our goal is to observe relatively long time-sample, covering both tranquil and turbulent periods. This approach gives us a more realistic downside risk measures, because when different crisis periods are included in the sample, then it produces higher Value-at-Risk measures. All time-series between Bitcoin and six different assets are synchronized according to the existing observations. Table 1 contains concise summary statistics, including first four moments, Jarque-Bera (JB) and Ljung-Box (LB) tests as well as Dickey-Fuller generalized least square (DF-GLS) unit root tests.

According to Table 1, Bitcoin has the highest mean, but also the highest standard deviation, which suggests that Bitcoin investment can produce relatively

[1] Due to usage of world wide assets, time zones may play a role sometimes, i.e. data in different markets may not be perfectly aligned in data repository. For instance, trading in Shanghai may start on 01/09/2020, but the Bitcoin time series may still be dated on 31/08/2020 in data repository. However, this discrepancy has negligible or no effect on final results, because we rerun our analysis with different time-stamps of Bitcoin, but the results are very similar with original results.

high yield for investors, but also at high risk cost. This implies that Bitcoin is a good candidate for diversification. Other six assets have lower risk comparing to Bitcoin, which makes them a favourable auxiliary asset in a portfolio with Bitcoin. Four out of seven assets are left skewed, while all selected asset have very high kurtosis, meaning that all assets significantly deviates from normal distribution. This is the reason why we use Student-t distribution in univariate GARCH models. All time-series, except corn, reports autocorrelation, whereas all selected assets have time-varying variance feature. These characteristics justify the usage of ARMA-GARCH models in DCC framework. Besides, none of the time-series reports the presence of unit root, which is a necessary precondition for GARCH modelling.

Table 1 Descriptive Statistics of the Selected Assets

	Mean	*St. dev.*	*Skew.*	*Kurt.*	*JB*	*LB(Q)*	*LB(Q^2)*	*DF-GLS*
Bitcoin	0.483	5.979	0.116	13.309	11961.1	0.000	0.000	-8.584
10Y bond	-0.003	2.909	0.999	27.632	68704.5	0.000	0.000	-8.416
Gold	0.088	1.419	1.000	19.986	32846.6	0.000	0.000	-11.857
Brent	-0.009	2.270	-0.989	25.002	55084.5	0.004	0.000	-15.109
S&P500	0.048	1.095	-0.913	20.700	34854.3	0.000	0.000	-7.735
SHCOMP	0.012	1.350	-0.943	9.474	4815.7	0.000	0.000	-5.230
Corn	0.011	1.568	-0.272	6.655	1504.9	0.852	0.000	-6.998

Notes: JB stands for value of Jarque-Bera coefficients of normality, LB(Q) and LB(Q^2) tests refer to p-values of Ljung-Box Q-statistics of level and squared residuals of 20 lags. 1% and 5% critical values for DF-GLS test with 5 lags, assuming only constant, are -2.566 and -1.941, respectively.

In order to be precise in dynamic correlation estimation, we combine four different GARCH specification (simple GARCH, GJRGARCH, EGARCH and NAGARCH) and three different multivariate distribution (normal, Student and Laplace). DCC model with the lowest AIC value is the optimal one. Table 2 contains these values.

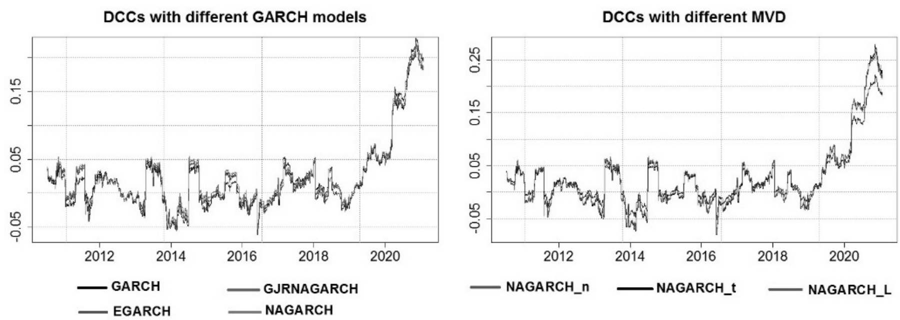
Table 2 Calculated AIC Values for Different DCC Models

	Bitcoin vs bond	*Bitcoin vs gold*	*Bitcoin vs Brent*	*Bitcoin vs S&P500*	*Bitcoin vs SHCOMP*	*Bitcoin vs corn*
DCC-GARCH_n	10.485	9.460	10.066	8.469	9.200	9.654
DCC-GJRGARCH_n	10.474	9.449	10.037	8.437	9.198	9.653
DCC-EGARCH_n	10.479	9.448	10.026	8.421	9.194	9.642
DCC-NAGARCH_n	10.472	9.445	10.027	8.403	9.197	9.652
	Estimated optimal DCC models with different multivariate distributions					
	NAGARCH	*NAGARCH*	*EGARCH*	*NAGARCH*	*EGARCH*	*EGARCH*
Student t	10.258	8.779	9.735	8.109	8.858	9.337
Laplace	10.316	8.761	9.770	8.140	8.855	9.365

Notes: Greyed values indicate the lowest AIC.

Table 2 shows that three times EGARCH and NAGARCH combined with MVD normal have an upper hand. When these models are estimated with MVD Student t and Laplace, then DCC with MVD Student t has precedence 4 times, and with MVD Laplace two times. In order to justify the usage of different DCC specifications, we show two plots in Figure 2, which present estimated DCCs with different GARCH models and with different MVD. It can be noticed that tiny distinctions exist among plotted dynamic correlations, which might contribute to the accuracy of the created portfolios.

Figure 2 DCCs of Bitcoin-Gold Estimated with Different GARCH Models and MVD



5. Research Results

5.1 Estimated Dynamic Correlations

This section presents the results of DCC models and estimated dynamic correlations. In order to be concise in result presentation, we only show estimated multivariate parameters in Table 3, while the results for univariate GARCH models can be obtained by request. All estimated dynamic correlations are presented in Figure 3. According to Table 3, all b parameters are highly statistically significant. These parameters capture the persistence of dynamic correlation and their statistical significance is crucial for the reliability of DCCs. Two out of four a parameters are significant, and these coefficients measure an ARCH effect in DCCs. However, a parameters do not have a vital role in the estimation of DCCs, so their insignificance is not particularly important for trustworthiness of dynamic correlations.

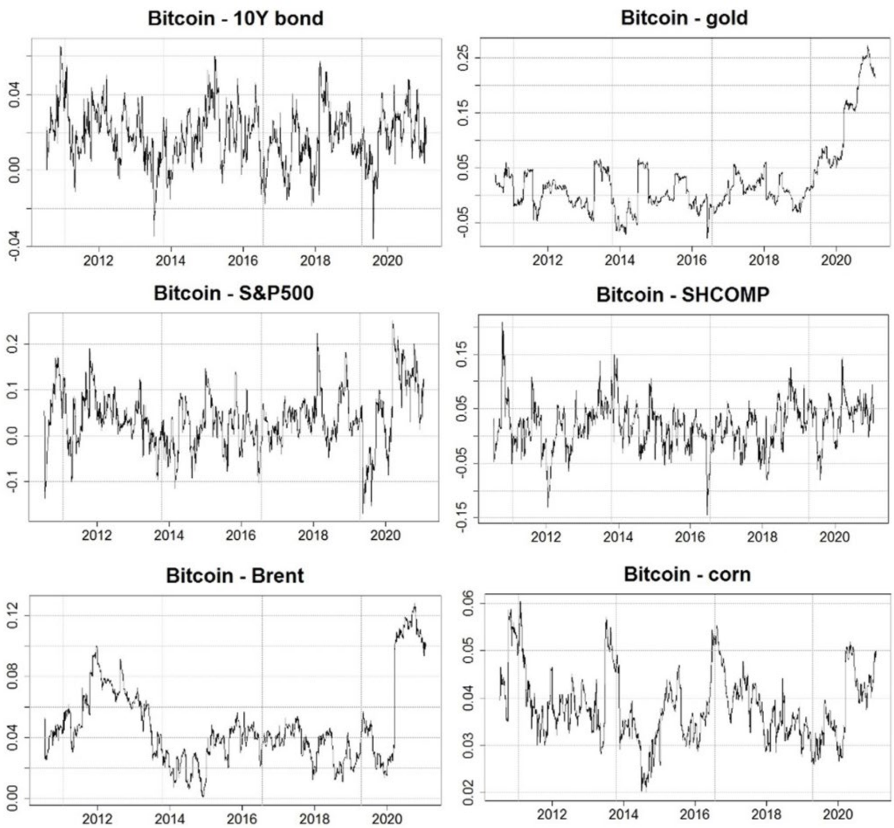
Table 3 DCC Estimates and Average Dynamic Correlations

Estimated DCC parameters						
	Bitcoin vs bond	***Bitcoin vs gold***	***Bitcoin vs Brent***	***Bitcoin vs S&P500***	***Bitcoin vs SHCOMP***	***Bitcoin vs corn***
a	0.004	0.006^{***}	0.002	0.016^{**}	0.001	0.002
b	0.944^{***}	0.993^{***}	0.995^{***}	0.953^{***}	0.987^{***}	0.985^{***}
M-shape	5.18^{***}	–	4.25^{***}	4.37^{***}	4.20^{***}	–
Average ρ	0.018	0.025	0.047	0.038	0.021	0.038

Notes: ***, ** represent statistical significance at the 1%, 5% and 10% level, respectively.

In addition, all MVD (M-shape) parameters are highly statistically significant, which indicates that chosen Student t distribution accurately describes distribution of dynamic correlation. Table 3 also presents the average values of dynamic correlations, while Figure 3 shows how DCCs evolve over time. Having a knowledge of average dynamic correlations, we can determine whether particular asset have properties of diversifier, hedge or safe haven. Baur and Lucey (2010) explained what is the difference between these terms. If an asset is positively (but not perfectly correlated) with another asset or portfolio on average, such asset can be classified as diversifier. Hedge explains an asset that is uncorrelated or negatively correlated with another asset or portfolio on average, while safe haven is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio in periods of market downturn.

Figure 3 Estimated DCCs between Bitcoin and Six Selected Assets



As can be seen, all dynamic correlations are on average very weak, which very well coincide with the papers that reported low correlations between cryptocurrencies and traditional assets (see e.g. Brière et al., 2015; Dewandaru et al., 2015; Corbet et al., 2018). According to Figure 3, dynamic correlations for bond, Brent and corn are mostly above zero, which suggests that these assets can only be regarded as diversifier. On the other hand, gold and particularly two indices – S&P500 and SHCOMP has conspicuous periods when DCCs go below zero, which indicates that these assets also can be viewed as a hedge. It can be seen that all DCCs do not have huge oscillations across time, while only gold and Brent have somewhat increased positive correlation with Bitcoin during ongoing COVID19 pandemic crisis. This is in line with the assertion of Baur et al., 2018 who claimed that Bitcoin and traditional assets (stocks, bonds and currencies) are correlated neither in financial turmoil nor in normal times.

Dynamic correlation results could indicate that all selected assets could reduce risk in combination with Bitcoin, but they cannot tell, based on visual inspection, which asset is the best risk-minimizer and what is actual measure of that. Next subsections will answer this question, since they present the results of downside risk reduction observed *via* four different risk metrics.

5.2 Parametric Downside Risk Measures for Portfolios

After the construction of the dynamic correlations, we calculate dynamic weights of auxiliary assets and construct six portfolios. These portfolios are subject to downside risk measurement, and this subsection presents the findings of parametric VaR and CVaR. Table 4 presents calculated weights of six assets, suggesting that S&P500 index has the highest weight, while SHCOMP and gold follow. Figure 4 presents actual dynamics of weights for six assets. However, dynamic weights, similarly as dynamic correlations, do not say nothing about risk-reducing performances of the assets. Table 6 serves for this purpose, i.e. it contains calculated parametric Value-at-Risk and conditional Value-at-Risk measures, along with the hedge effectiveness indices. Measures for minimum variance are also included, because all portfolio are minimum-variance by default. However, these results do not have any deeper meaning in our research, because minimum variances do not bear information about downside risk, since variance gives an equal weight to positive and negative returns.

Table 4 Calculated Weights of Auxiliary Asset in Two-Asset Portfolio with Bitcoin

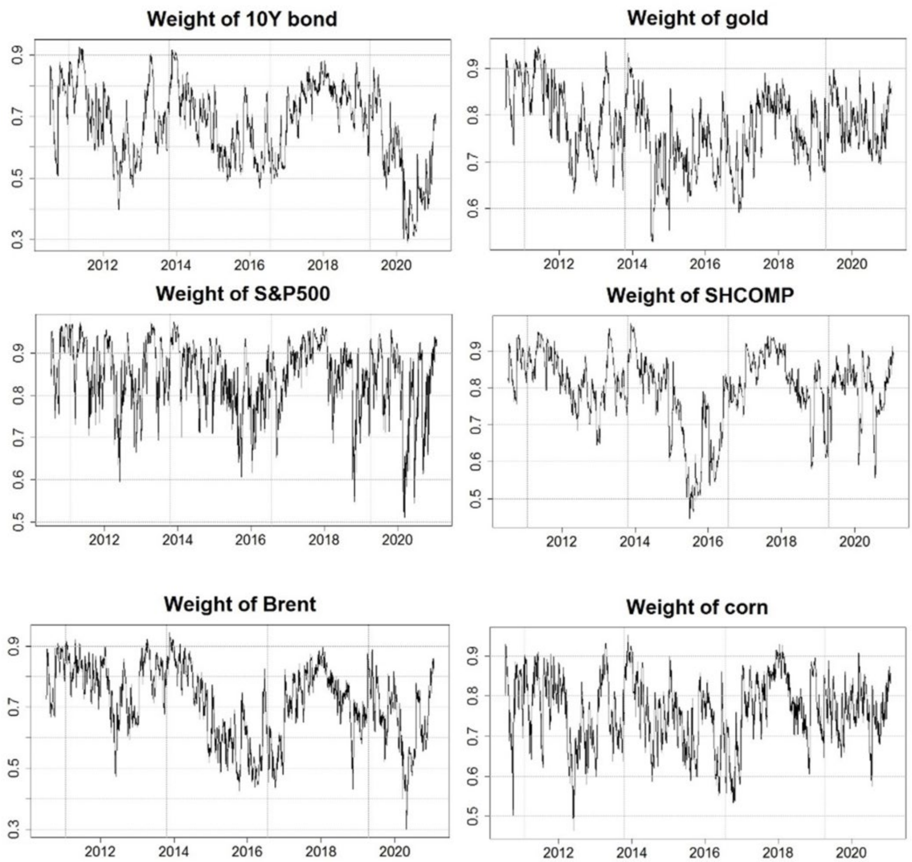
	Bitcoin vs bond	*Bitcoin vs gold*	*Bitcoin vs Brent*	*Bitcoin vs S&P500*	*Bitcoin vs SHCOMP*	*Bitcoin vs corn*
Weight	67%	77%	72%	85%	80%	76%

In addition, it is obvious that all VaR and CVaR values are calculated under somewhat unconventional probability intervals, i.e. 4%, 3.5%, 3%, 2% and 1%. The reason why we observe these downside risk measures only at relatively high probability levels lies in the fact that along VaR and CVaR, we also calculate semiparametric downside risk measures – mVaR and mCVaR. Following Cavenaile and Lejeune (2012)[2], we learned that mVaR can be consistently used only over a limited interval of confidence levels. These authors contended that under 95.84% confidence level, modified Value-at-Risk should never be used, while the use of

[2] The authors based their conclusions on simulation.

higher confidence levels is limited by the value of skewness. If these restrictions are disregarded, then modified Value-at-Risk will be mistakenly assessed. These restrictions do not apply for an ordinary VaR, but since we want to compare the results of VaR and mVaR, we also restrict calculation of VaR to aforementioned lower confidence levels.

Figure 4 Calculated Dynamic Weights of the Selected Auxiliary Assets



Looking at Table 5 results, portfolio with S&P500 index has the best risk-minimizing results. This finding applies for all types of risk measures, i.e. variance and two downside risk measured – Value-at-Risk and conditional Value-at-Risk. For instance, at probability level of 96%, VaR for Bitcoin-S&P500 amounts -2.175, which means that that there is a 4% chance that investor will lose 2.175% or more in value of portfolio in a single day. When probability is increased to 99%, minimal loss for investor rise to -2.914 in a single day. The second-best downside risk-minimizing asset is Chinese index SHCOMP, while gold is the third one. Portfolio with 10Y bond has by far the worst risk-minimizing result. According to the results, two best risk-minimizing portfolios are constructed with stock indices, which is in line with some other papers such as Baur et al. (2018), Dyhrberg (2016) and Fang et al. (2019). The explanation why stock indices have the best risk-minimizing performances lies in two facts. First, both stock indices have very low correlation with Bitcoin (see Table 3). However, this is not a decisive factor, because all other assets also have very low correlation with Bitcoin, even lower. More important feature of an auxiliary asset is the level of its risk. Khalfaoui et al. (2015) explained that the instrument with the lowest risk produces the best risk-minimizing results in a portfolio with primary instrument. According to this claim, Table 1 suggests that S&P500 and SHCOMP are two assets with the lowest standard deviation which perfectly fits in with the VaR and CVaR results and with the assertion of Khalfaoui et al. (2015).

Besides VaR, we also calculate CVaR, which complements VaR and correct certain deficiency that is characteristic for VaR, i.e. in conditions when potential loss, calculated by VaR, is exceeded. As can be seen, CVaR results are higher than VaR for every risk level that is considered, and it applies for every portfolio. This is expected, because CVaR measures an average expected loss rather than a range of potential losses that VaR provides. For instance, CVaR for Bitcoin-S&P500 is -2.693 under 96% probability, which means that in the worst 4% of returns, the average loss will be 2.693%, while VaR at the same probability level is -2.175. The same consistency is maintained throughout all portfolios. Also, Figure 5 illustrates that calculated VaR and CVaR results are elegantly lined up one under the other, i.e. without overlapping, regarding all constructed portfolios and five different probability levels. This suggests that no matter how investor is repulsive towards risk, Bitcoin combination with S&P500 index will always yield the best risk-minimizing downside risk measures, while SHCOMP and gold will follow.

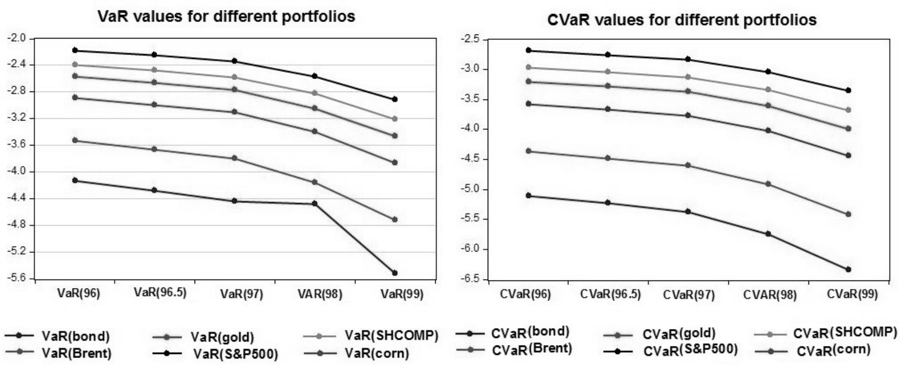
Table 5 Downside Risk Results Observed Via Parametric VaR and CVaR

		Bitcoin vs bond	*Bitcoin vs gold*	*Bitcoin vs Brent*	*Bitcoin vs S&P500*	*Bitcoin vs SHCOMP*	*Bitcoin vs corn*
Panel A: Risk measures							
Var	–	5.853	2.405	4.264	1.647	1.996	2.896
VaR	4%	-4.131	-2.574	-3.535	-2.175	-2.394	-2.888
	3.5%	-4.279	-2.669	-3.661	-2.254	-2.481	-2.992
	3%	-4.446	-2.776	-3.804	-2.342	-2.578	-3.109
	2%	-4.484	-3.044	-4.161	-2.564	-2.823	-3.403
	1%	-5.523	-3.467	-4.724	-2.914	-3.208	-3.867
CVaR	4%	-5.107	-3.200	-4.368	-2.693	-2.965	-3.574
	3.5%	-5.236	-3.283	-4.479	-2.762	-3.04	-3.665
	3%	-5.382	-3.376	-4.603	-2.839	-3.125	-3.768
	2%	-5.752	-3.613	-4.919	-3.036	-3.341	-4.028
	1%	-6.343	-3.992	-5.423	-3.349	-3.686	-4.444
Panel B: Hedge effectiveness indices							
HEI_{Var}		0.836	0.932	0.880	0.954	0.945	0.919
HEI_{VaR}		0.587	0.741	0.645	0.782	0.764	0.712
HEI_{CVaR}		0.588	0.741	0.647	0.783	0.764	0.712

Note: Greyed HEI values indicate the highest HEI.

Panel B in Table 5 presents the results of hedge effectiveness calculated for only 96% probability, because HEI values for all other probabilities are very similar. HEI values mirror calculated VAR and CVaR metrices, and as can be seen, portfolio with S&P500 has the highest all three HEI values. In other words, comparing to sole investment in Bitcoin, portfolio with S&P500 lowers variance by 95%, and both downside risks by 78%. This clearly indicates that Bitcoin-S&P500 combination lowers risk the most.

Figure 5 Illustrative Presentation of Calculated VaR and CVaR Metrices



According to the presented result, S&P500 is a clear winner, regarding both downside risk measures. However, it should not be forgotten that parametric VaR and CVaR perform under relatively strict assumption that portfolio follow Gaussian distribution. This is very bold hypothesis, having in mind that we deal with daily financial time-series, which are frequently characterized by heavy tails and negative skewness. Therefore, it is very likely that the presented results are biased and erroneous. In order to check this assumption, we additionally take an alternative approach that is based on Cornish-Fisher expansion, which overcomes this drawback of parametric VaR and CVaR. As have been said, this method is known as modified VaR and CVaR, and it accounts non-Gaussian characteristics of a portfolio, i.e. the higher moments – skewness and kurtosis. Next subsection presents these results.

5.3 Semiparametric Downside Risk Measures for Portfolios

Modified VaR improves parametric VaR by rewarding low kurtosis and positive skewness and penalizing high kurtosis and negative skewness. However, if we want to calculate properly modified downside risk measures, we have to be sure that these results are reliable. As mentioned earlier, Cavenaile and Lejeune (2012) asserted that modified VaR can be consistent only over a limited interval of confidence levels. They claimed that under 95.84% confidence level, modified Value-at-Risk should never be calculated, while the use of higher confidence levels is conditional on the value of skewness. Table 6 contains minimum skewness that restricts upper confidence level in mVaR calculation.

In order to make a preliminary insight whether ordinary VaR and CVaR are biased, and whether mVaR calculation is necessary after all, we present Table 7, which contains stylized facts regarding six constructed portfolios.

Table 6 Minimum Skewness for mVaR Consistency

Confidence level	96.0%	97.5%	99.0%	99,5%	99.9%
Minimum skewness	-3.3	-1.62	-0.98	-0.79	-0.59

Source: Cavenaile and Lejeune (2012)

As can be seen, five out of six portfolios have negative skewness, while all portfolios have high kurtosis, whereby portfolio with S&P500 has the highest one. These portfolios' features clearly indicate that none of the portfolios has Gaussian distribution, which means that classical downside risk measures are probably misleading. This gives us a confidence to pursue with non-Gaussian VaR and CVaR calculations.

Table 7 Descriptive Statistics of the Created Portfolios

	Mean	***St. dev.***	***Skew.***	***Kurt.***
Bitcoin-bond	0.105	5.979	-0.378	12.165
Bitcoin-gold	0.141	1.551	0.048	7.227
Bitcoin-Brent	0.080	2.065	-1.645	13.697
Bitcoin-S&P500	0.071	1.283	-2.440	36.942
Bitcoin-SHCOMP	0.079	1.413	-0.372	5.143
Bitcoin-corn	0.092	1.701	-0.138	5.449

Our decision to use semiparametric measures is additionally backed by the Kupiec Likelihood Ratio (LR) test (Kupiec, 1995), which can give us an insight about the performance of the considered parametric VaR model. Using this test, we calculate the empirical failure rate for the left tail (short position) of the return distributions. The failure rate is defined as a number of times the portfolio return series exceeds the estimated value of VaR. If the failure rate (f) is equal to the pre-specified VaR level (α), then the associated VaR model is correctly specified. On the contrary, VaR underestimates risk. Equation (17) explains how the Kupiec Likelihood Ratio (LR) test is calculated.

$$LR = -2ln[\alpha^{T-N}(1-\alpha)^{N}] + 2ln[(1-f)^{T-N}f^{T}], \tag{17}$$

where f stands for the empirical failure rate calculated as the ratio of the number of return observations (N) that exceed the estimated VaR in the sample of (T) observations. α denotes the pre-specified VaR level, which is in our case 96%, 96.5%, 97%, 98% and 99%. The Kupiec LR statistic is asymptotically chi-squared distributed with one level of freedom ($\chi^2_{(1)}$). The null hypothesis states that calculated LR value asymptotically converges to a chi squared distribution with one degree of freedom. If the observed value for LR exceeds the critical value of $\chi^2_{(1)}$ at the present level of significance a, then the null hypothesis is rejected. For the calculation of Kupiec test, we use fitted values obtained from the simplest AR(1)-GARCH-normal model. In this way, we generate portfolio time-series that are $i.i.d.$, which is an important precondition for calculating Kupiec test (see Katris and Daskalaki, 2014). Table 8 presents these results. According to Table 8, in all portfolio cases, we find some test values suggesting that the portfolio return series exceeds the estimated value of VaR. Most failures we find in portfolios with Brent and S&P500 index, and these two portfolios reported the highest kurtosis values (see Table 7). In other words, extreme values or outliers are present in all the portfolio return series, which indicates that semiparametric measures might be more appropriate.

Table 8 Kupiec Test Results

	Bitcoin vs bond	***Bitcoin vs gold***	***Bitcoin vs Brent***	***Bitcoin vs S&P500***	***Bitcoin vs SHCOMP***	***Bitcoin vs corn***
0.96%	1.974 (0.160)	0.769 (0.381)	6.993 (0.008)	13.354 (0.000)	2.006 (0.157)	0.991 (0.320)
0.965%	1.254 (0.263)	0.209 (0.648)	4.405 (0.0359)	14.355 (0.000)	0.969 (0.324)	0.293 (0.588)
0.97%	1.612 (0.204)	0.125 (0.723)	6.705 (0.010)	10.974 (0.000)	0.250 (0.617)	0.001 (0.976)
0.98%	0.173 (0.677)	5.618 (0.018)	2.415 (0.120)	2.439 (0.118)	0.014 (0.905)	0.338 (0.561)
0.99%	3.934 (0.047)	25.633 (0.000)	0.154 (0.695)	0.469 (0.493)	4.672 (0.031)	3.939 (0.047)

Notes: Greyed values denote cases when null hypothesis is rejected. p values are in parentheses. Critical value of $\chi^2_{(1)}$ test at the 99% (95%) confidence level is 6.64 (3.84).

Table 9 contains the results of semiparametric VaR and CVaR, while Figure 6 plots these results. Following Cavenaile and Lejeune (2012), we restrict an upper mVaR and mCvaR limit for Bitcoin-S&P500 and Bitcoin-Brent portfolios to 97% confidence level.

According to the results in Table 9, the order of the best and worst downside risk portfolios is shuffled in comparing to Table 5. In other words, S&P500 index is no longer the best auxiliary asset in combination with Bitcoin, from the aspect of the lowest downside risks. As a matter of fact, S&P500 index is one of the worst instruments to be combined with Bitcoin, taking into account both downside risk measures. Figure 6 illustratively shows that only Brent and the U.S. bond have worse results than S&P500 index from the aspect of mVaR, whereas from the point of mCVaR, S&P500 index is actually the worst performing asset in a portfolio with Bitcoin. These findings undoubtedly confirm that higher moments have very important role when it comes to the measurement of downside risks.

On the other hand, gold improved its position, and now it is the second-best instrument, regarding both downside risk measures, while gold shares first place with SHCOMP index for mVaR under 96% probability level. Gold owns these good results to positive skewness and relatively low kurtosis, as Table 7 shows. Corn also improved its position, and moved from fourth to third place, due to relatively low kurtosis and negative skewness. However, the instrument that lowers the most downside risks is Chinese index SHCOMP, and its good performance lies in the lowest kurtosis and relatively low negative skewness.

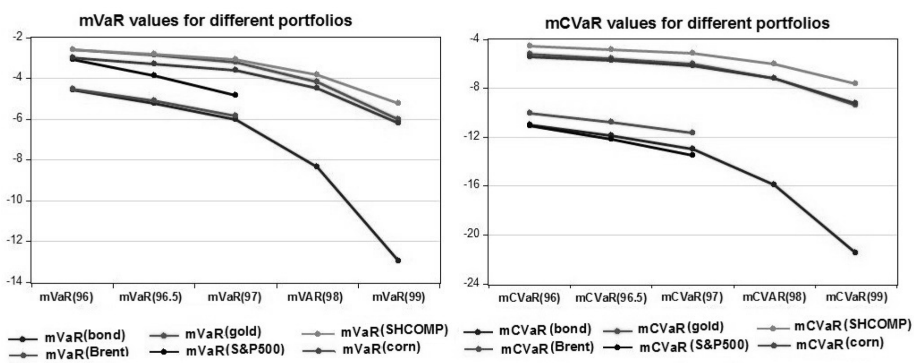
Table 9 Downside Risk Results Observed Via Semiparametric VaR and CVaR

		Bitcoin vs bond	*Bitcoin vs gold*	*Bitcoin vs Brent*	*Bitcoin vs S&P500*	*Bitcoin vs SHCOMP*	*Bitcoin vs corn*
Panel A: Modified risk measures							
mVaR	4%	-4.566	-2.601	-4.531	-3.058	-2.599	-3.010
	3,5%	-5.229	-2.879	-5.118	-3.856	-2.821	-3.276
	3%	-6.034	-3.215	-5.826	-4.835	-3.085	-3.595
	2%	-8.354	-4.171	NA	NA	-3.823	-4.489
	1%	-12.947	-6.03	NA	NA	-5.219	-6.195
mCVaR	4%	-11.032	-5.229	-10.031	-11.104	-4.582	-5.431
	3,5%	-11.909	-5.585	-10.776	-12.198	-4.85	-5.758
	3%	-12.958	-6.009	-11.662	-13.51	-5.167	-6.146
	2%	-15.891	-7.187	NA	NA	-6.038	-7.216
	1%	-21.457	-9.402	NA	NA	-7.648	-9.205
Panel B: Hedge effectiveness indices for modified risk metrics							
HEI_{mVaR} *(96%)*		0.545	0.740	0.599	0.696	0.746	0.701
HEI_{mCVaR} *(96%)*		0.528	0.777	0.559	0.531	0.809	0.771
HEI_{mVaR} *(97%)*		0.538	0.754	0.584	0.632	0.768	0.726
HEI_{mCVaR} *(97%)*		0.526	0.781	0.554	0.513	0.815	0.778
HEI_{mVaR} *(99%)*		0.526	0.781	NA	NA	0.813	0.776
HEI_{mCVaR} *(99%)*		0.523	0.792	NA	NA	0.833	0.798

Notes: Greyed HEI values indicate the highest HEI.

According to Figure 6, SHCOMP is slightly better than gold from the aspect of mVaR, whereas from the conditional mVaR, SHCOMP index has clearly the best results, which means that at all probability levels, SHCOMP index has the lowest average loss. Our results are in line with the paper of Guesmi et al. (2019) who asserted that hedging strategies involving gold, oil, equities and Bitcoin reduce considerably the portfolio's risk.

Figure 6 Illustrative Presentation of Calculated mVaR and mCVaR Metrices



At the end, we comment HEI values. Unlike Table 5, in Table 9, we show HEI numbers at different probabilities, because HEI values for modified VaR and CVaR have a tendency to differ between various probability levels. For the sake of conciseness, we calculate HEI values for 96%, 97% and 99%. It can be noticed that for some portfolios HEI numbers increase (decrease) when probabilities rise. In particular, for bond, Brent and S&P500 (the three worst assets) HEI numbers lowers with the increase of probability. This implies that mVaR and mCVaR metrices rise faster than Bitcoin counterpart with the increase of probabilities. On the other hand, for SHCOMP, gold and corn (the three best assets), HEI numbers ger higher with the increase of probability, which indicates that both downside risk measures rise slower than the Bitcoin counterpart.

Although investors worry about risk, they are also interested in high returns. Therefore, we calculate Sharpe ratio that takes into account both returns and risk. In particular, Sharpe ratio calculates average return earned in excess of the risk-free rate per unit of risk or standard deviation, which is shown in equation (18). Basically, it indicates how investor is rewarded per unit of risk that he takes. Investment is better if Sharpe ratio is higher and *vice versa*. Table 10 contains calculated Sharpe ratios of six portfolios, but also for sole investment in Bitcoin.

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p} \tag{18}$$

where R_p is an average return of portfolio, R_f is risk-free rate, and σ_p is standard deviation of portfolio. For risk-free rate, we take yields of 3M treasury bills.

According to Table 10, by far the highest Sharpe ratio has portfolio with gold, the second best is portfolio with SHCOMP index, while portfolios with S&P500 and corn are in the middle. The worst investments from return/risk aspect are portfolios with Brent and bond. This means that portfolio with gold has the best relation between high returns and risk. In other words, investors who combine Bitcoin with gold can realise the highest returns, taking relatively low level of risk. Although portfolio with SHCOMP has lower risk comparing to portfolio with gold, it also has significantly lower average returns, and this is why Sharpe ratio of former portfolio is significantly lower comparing to the latter. Kajtazi and Moro (2019) reported that adding bitcoin in portfolio can improve its performance, but this comes more from the increase in returns than in the reduction of volatility.

Table 10 Calculated Sharpe Ratio

	Only Bitcoin	*Bitcoin vs bond*	*Bitcoin vs gold*	*Bitcoin vs Brent*	*Bitcoin vs S&P500*	*Bitcoin vs SHCOMP*	*Bitcoin vs corn*
Sharpe ratio	0.07432	0.02964	0.09179	0.03027	0.04666	0.05116	0.04373

In addition, Table 10 shows an interesting finding that undiversified investment in Bitcoin has very high Sharpe ratio, which might indicate that sole investment in Bitcoin is a good investment. However, this can be deceiving. This is because investment in Bitcoin enjoys high returns, as a matter of fact, much higher than all portfolios' returns, but these high returns come with an excess of additional risk. Therefore, a sole investment in Bitcoin is not smart decision for risk-averse investors, because it bears a lot of extra risk. Only investors who are sympathetic with high risk can invest in bitcoin without diversification, but this enters a realm of speculation.

6. Conclusions

This paper tries to find out which auxiliary instrument – S&P500, SHCOMP, the U.S. 10Y bond, gold, Brent oil and corn, produces the best downside risk results in a combination with Bitcoin. Before construction of six portfolios, we estimate the optimal DCC model for every pair of assets, taking care of the best univariate GARCH model and MVD in DCC framework. Risk performance is measured *via* parametric VaR and CVaR and their modified versions, i.e. semiparametric VaR and CVaR metrices.

We report several interesting findings. First, all auxiliary assets have very low dynamic correlation with Bitcoin, which is an important precondition that these instruments can be regarded as diversifiers. We intentionally calculate parametric and semiparametric VaR and CVaR metrices at different probability levels, because we want to compare them. Reason behind this approach lies in a fact that former method assumes normal distribution of portfolio, which is a very strict assumption, while the latter method accounts higher moments in downside risk calculation. According to parametric measures, S&P500 has the best risk-minimizing results, regarding both downside risk metrices. SHCOMP is the second-best asset, while gold and corn take third and fourth place.

However, when non-normal features of portfolios are taken into account, situation changes dramatically. More specifically, S&P500 loses primacy and falls to the rear, while SHCOMP and gold ascend to first and second place, respectively. Explanation for these results lies in a fact that modified VaR and CVaR favour low kurtosis and positive skewness, which are completely opposite attributes that S&P500 index has. On the other hand, these favourable qualities are characteristics of SHCOMP and gold, and these are the main causes why SHCOMP and gold emerged as the most suitable assets that lowers downside risks in the best way. We also calculate Sharpe ratio, which suggests that portfolio with gold has by far the best return/risk characteristics.

This paper has an important message for investors who intend to pursue a minimal downside risks, regarding investments not only in Bitcoin, but generally. In particular, the results clearly show that parametric downside risk measures can be misleading if higher moments of portfolios are not taken into account, which unequivocally gives an upper hand to semiparametric risk measures. As for those investors who want to minimize losses in portfolios with Bitcoin, the paper recommends combination with Chinese SHCOMP index.

Future studies can extend this research by taking another approach of VaR modelling. For instance, liquidity adjusted VaR is based on the assumption that not all assets are equally liquid, whereby investors' liquidation of their assets can cause a significant price change. It would be interesting to see how VaR performs if liquidity risk is incorporated in its calculation.

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