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JEDNOFAKTORSKI MODEL ZA STOPE NEIZMIRENJA PO TIPOVIMA KREDITA

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Rezime

U radu se istražuje veza između stopa neizmirenja po tipovima kredita i sistemske komponente kreditnog rizika. Ova veza je opisana linearnim modelom koji kombinuje uticaj sistemske i idiosinkratske komponente kreditnog rizika. Sistemska komponenta je latentni faktor koji direktno zavisi od agregatne stope neizmirenja svih kredita, dok idiosinkratska komponenta utiče na specifične varijacije stope neizmirenja za pojedinačne tipove kredita. Transformacijom opservabilnih veličina model se ekonometrijski može predstaviti kao mešoviti linearni model, a sistemska i idiosinkratska komponente predstavljaju, redom, nagib i odsečak koji su specifični za svaki tip kredita ponaosob. Predloženi model je ilustrovan na panelu podataka o plasmanima u statusu neizmirenja Udruženja banaka Srbije. Dobijeni rezultati pokazuju veoma visoku moć modela da objasni prosečne stope neizmirenja svih tipova kredita. Agregatna stopa neizmirenja stoga igra ulogu jedinstvene sistemske komponente koja jednostavno oponaša uticaje stvarnih makroekonomskih faktora rizika bez potrebe za eksplicitnim modeliranjem ove veze.

Ključne reči: kreditni rizik, stopa neizmirenja, sistemski faktori, mešoviti linearni model

JEL klasifikacija: G21, C23.

Uvod

Dekompozicija rizika na faktore koji opisuju njegove različite izvore predstavlja jedno od fundamentalnih pitanja finansijske ekonomije. Doprinosi pojedinačnih komponenata rizika su značajni jer omogućavaju praćenje izvora rizika koji mogu dovesti do gubitaka u portfolijima. Prepoznavanje izvora rizika je posebno značajno za kompleksne portfolije finansijskih institucija, kao i određene vrste instrumenata gde faktori nisu manifestno prepoznatljivi, poput nekih tipova derivata.

Međutim, standardna podela komponenti rizika na sistemsku i idiosinkratsku komponentu (Bodie *et al.,* 2021) implicitno podrazumeva prevashodno cenovne rizike instrumenata na tržištima hartija od vrednosti. Relativno manje pažnje je do sada posvećeno analognoj dekompoziciji kreditnog rizika, koji je svakako najvažnija vrsta rizika za poslovne banke i druge kreditne institucije. Ovakav pristup se umnogome promenio nakon globalne finansijske krize, koja je nedvosmisleno pokazala da kod upravljanja kreditnim rizikom portfolija ne možemo posmatrati samo faktore specifične za dužnika. Potrebno je uzeti u obzir moguće efekte prelivanja rizika pojedinačnih dužnika na čitave sektore privrede, pa čak i na privredu u celini (Ballester *et al.*, 2016). Ova prelivanja možemo (bar na apstraktnom nivou) razumeti upravo kroz sistemsku komponentu kreditnog rizika, koja je zajednička za sve tipove dužnika pa samim tim i za čitave portfolije banaka.

Sistemska komponenta potiče od makroekonomskih faktora rizika i po tome je sasvim analogna sa tržišnim rizikom. Za finansijsku stabilnost, koja je postala jedno od centralnih pitanja u postkriznim regulatornim okvirima (BCBS, 2011), od posebnog su značaja uticaji sistemskih faktora na finansijski i vladin sektor (Acharya *et al.,* 2014), kao i kanal prelivanja sa ovih na nefinansijske sektore (Gross & Siklos, 2020). Ipak, na praktičnom nivou ovi faktori se u bazelskom regulatornom okviru uglavnom ne modeliraju eksplicitno unutar prvog stuba,[1] već se njima upravlja posredno, kroz kontraciklične zaštitne slojeve kapitala i rizik koncentracije. Repullo & Saurina (2011) su kritikovali mehaničku primenu kontracikličnih zaštitnih slojeva i argumentovali da pojačava inherentnu procikličnost regulatornih zahteva za kapitalom.

Strategije upravljanja kreditnim rizikom finansijskih institucija moraju uzeti u obzir poslovne cikluse i povezane promene u kvalitetu i sastavu kreditnog portfolija (Jiménez & Saurina, 2006). Da bi ove strategije bile održive na duži rok, finansijske institucije moraju pratiti osetljivost svojih portfolija na promene različitih makroekonomskih i finansijskih indikatora. Stoga pitanje stepena u kome pojedini segmenti privrede doprinose sistemskom riziku postaje jedno od ključnih (Novales & Chamizo, 2019). Možemo ga u određenoj meri poistovetiti sa izloženošću pojedinih delova kreditnog portfolija finansijske institucije sistemskoj komponenti rizika.

Literatura na temu dekompozicije kreditnog rizika na faktore je mahom novijeg datuma. Rosen & Saunders (2010) su istakli da je jedan od osnovnih izazova vezanih za ovu temu nelinearan uticaj faktora na opservabilne veličine. Oni su predložili višefaktorski model koji se Hoeffdingovom dekompozicijom vezuje za ukupne gubitke portfolija. Faktori koje su koristili su predstavljali kombinaciju makroekonomskih, geografskih, sektorskih i tržišnih faktora (kamatnih stopa, valutnih kurseva, volatilnosti prinosa akcija). Chamizo & Novales (2016) su koristili kreditne derivate (CDS), kako bi iz tržišnih podataka o

[1] Izuzetak predstavlja pristup zasnovan na internim modelima kreditnog rejtinga (engl. Internal Ratings Based [Approach], skraćeno IRB), gde se sistemska komponenta posredno modelira „rizikom portfolija". Ovaj faktor u IRB pristupu uvek uzima fiksnu vrednost u formuli za kapital dobijenoj na osnovu jednofaktorske Gausove kopule (Vasiček, 2002).

njihovim premijama razdvojili uticaj sistemske od idiosinkratske komponente kreditnog rizika velikih evropskih kompanija. Nešto ranije, determinante premija CDS ugovora proučavali su Ericsson *et al.* (2009). Koristeći pristup analize glavnih komponenti, oni su uspeli da identifikuju jedan dominantan zajednički faktor za sve kreditne izloženosti u uzorku. Rezidualni doprinos nesistemskih komponenata se najverovatnije može pripisati kombinaciji sektorskog i specifičnog faktora rizika (Bhansali *et al.*, 2008; Novales & Chamizo, 2019).

Povezanim pitanjem dekompozicije rizika druge ugovorne strane bavili su se de Graaf *et al.* (2018). Oni su dodatno ukazali na značaj razlaganja komponenata na što manji broj faktora zbog jednostavnijeg praćenja i interpretacije. Campbell *et al.* (2001) su proučavali dinamiku sistemske komponente tržišnog rizika i pokazali da ona ima ponašanje „vodećeg indikatora" u odnosu na idiosinkratske komponente. U kontekstu kreditnog rizika, u postojećoj literaturi se dinamika sistemske komponente najčešće procenjuje iz podataka sa tržišta obveznica i kreditnih derivata, ili se direktno povezuje sa makroekonomskim fundamentima. U akademskim istraživanjima relativno malo pažnje je do sada posvećeno dekompoziciji faktora rizika kreditnih portfolija poslovnih banaka i drugih nelikvidnih finansijskih instrumenata.

Prepoznajući očigledan istraživački jaz u literaturi, ovaj rad ima za cilj da predloži jednostavan i implementabilan model za dekompoziciju determinanti kreditnog rizika. Sledeći opštu logiku faktorskih modela vrednovanja finansijskih instrumenata, razvili smo linearan model sa jednim faktorom sistemskog rizika. Iako je u modelu taj faktor latentan, vezan je jednostavnom algebarskom transformacijom za opservabilnu stopu neizmirenja svih datih kredita u Srbiji. Rezidualni doprinos ukupnom riziku instrumenta ili portfolija pripisuje se komponenti rizika specifičnoj za tip kredita. Ilustracija modela je data na panel podacima Udruženja banaka Srbije (UBS) o plasmanima u statusu neizmirenja.

Ostatak rada organizovan je na sledeći način. U narednom odeljku izložen je teorijski model dekompozicije faktora kreditnog rizika na sistemsku i idiosinkratsku komponentu. U trećem odeljku su opisani podaci koji su korišćeni za ilustraciju modela, koja je prikazana u četvrtom odeljku. Poslednji odeljak sadrži zaključna razmatranja.

Model

Predloženi model razdvaja kreditni rizik pojedinačnih grupa (tipova) kredita na doprinose koji potiču od sistemske i specifične komponente rizika. Logika modela je donekle slična faktorskim modelima vrednovanja kapitala. Konkretno, pretpostavićemo da se kreditna sposobnost reprezentativnog dužnika unutar svake grupe i može opisati latentnim faktorom y. Dinamika promena ovog faktora data je sledećom linearnom vezom:

$$y_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it}. \tag{1}$$

U jednakosti (1), y_{it} je vrednost indeksa kreditne sposobnosti kategorije dužnika i u trenutku t, x_t je latentni indeks kreditne sposobnosti svih dužnika (tj. svih tipova kredita posmatranih agregirano), dok ε_{it} meri slučajno odstupanje od linearne veze. Koeficijenti α_i i β_i su specifični za svaku grupu dužnika, i predstavljaju mere idiosinkratskog i sistemskog rizika u svakoj grupi. Geometrijski se mogu interpretirati kao pojedinačni odsečci i nagibi u linearnoj regresiji po grupama. Konkretnije, pretpostavićemo da se oba mogu prikazati preko odstupanja od svojih srednjih vrednosti:

$$\alpha_i = \bar{\alpha} + u_i$$
$$\beta_i = \bar{\beta} + v_i. \quad (2)$$

Odstupanja u_i i v_i su pojedinačno nezavisna i potiču iz istih tipova raspodela slučajnih promenljivih, uz mogućnost postojanja korelacije između njih. Asimptotski ih možemo posmatrati i kao elemente dvodimenzionog slučajnog vektora iz zajedničke normalne raspodele:

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim \mathbf{\Phi}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}\right), \quad (3)$$

gde je $\mathbf{\Phi}: \mathbb{R}^2 \to [0, 1]$ funkcija normalne raspodele opisana odgovarajućim vektorom srednjih vrednosti i matrice kovarijansi.

Budući da su y_{it} i x_t latentni faktori, moramo ih povezati sa opservabilnim veličinama. Konkretno, pretpostavićemo da su vezani sa stopama neizmirenja na sledeći način:

$$d_{it} = \mathbb{P}(y_{it} < \mathbb{E}(y_{it}|x_t)|x_t)$$
$$D_t = \mathbb{P}(x_t < \mathbb{E}(x_t)), \quad (4)$$

gde $\mathbb{P}$ označava verovatnoću. Učestalost neizmirenja za grupu kredita i u trenutku t merimo odgovarajućom stopom neizmirenja d_{it}, a učestalost neizmirenja u trenutku t na nivou svih grupa merimo agregatnom stopom neizmirenja D_t. Relacije (4) nam govore da uspostavljamo jednakosti između (uslovne) verovatnoće da latentni faktori budu manji od svojih očekivanih vrednosti i stopa neizmirenja kao uzoračkih analogona ovih verovatnoća. Uz standardnu pretpostavku linearnih modela da su slučajne greške ε_{it} nezavisne i identično raspodeljene slučajne promenljive srednje vrednosti nula i varijanse σ_ε^2, ortogonalne na faktor x_t, lako je pokazati da je veličina

$$y_{it} - \mathbb{E}(y_{it}|x_t) = u_i + v_i x_t + \varepsilon_{it} \quad (5)$$

asimptotski normalno raspodeljena, te u podacima možemo koristiti vezu

$$y_{it} = G(d_{it}), \quad (6)$$

gde je $G: [0, 1] \to \mathbb{R}$ inverzna funkcija jednodimenzione normalne raspodele. Po analogiji ćemo koristiti sličnu vezu i za agregatne stope neizmirenja:

$$x_t = G(D_t). \quad (7)$$

Podaci

Za ilustraciju modela koristićemo agregirane anonimizovane podatke UBS o broju i iznosu plasmana u statusu neizmirenja. Podaci su granulirani po tipu kredita, tako da ima ukupno deset različitih tipova. Frekvencija podataka je kvartalna i obuhvata period između poslednjeg kvartala 2012. godine i poslednjeg kvartala 2018. godine. Stope neizmirenja računate su kao odnos ukupnih novčanih iznosa u statusu neizmirenja i ukupnih iznosa svih datih kredita. Logika ovog izbora detaljnije je objašnjena u nekim prethodnim radovima (videti, recimo, Fridson, 1991 ili Božović, 2019).

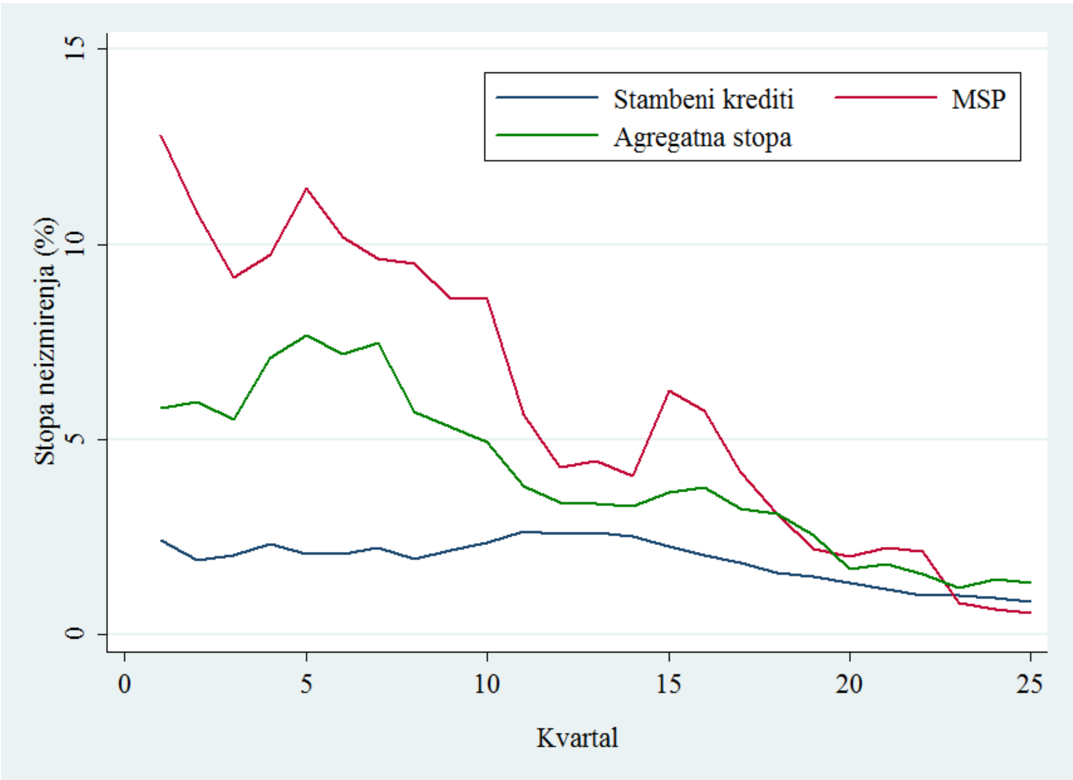
Deskriptivne statistike podataka prikazane su u Tabeli 1. Tabela sadrži informacije o srednjim vrednostima, standardnim devijacijama, minimumima i maksimumima stopa neizmirenja po tipovima proizvoda. Podela kredita po tipovima proizvoda izvršena je prema kategorizaciji u bazi UBS. U poslednjem redu tabele (oznaka „Ukupno“) prikazani su podaci za sve banke u uzorku, tj. podaci dobijeni na osnovu agregatnih stopa neizmirenja. U tabeli lako možemo uočiti jednu pravilnost u podacima: veće prosečne stope neizmirenja idu u paru sa većim standardnim devijacijama ovih stopa. Intuicija za ovakvu pojavu je slična onoj kod tržišnih instrumenata, gde veća prosečna stopa promene prinosa potiče od većeg rizika merenog volatilnošću prinosa. Naravno, ovu analogiju ne treba shvatiti doslovno. Osnovna ideja modela koji prikazujemo u radu jeste da poveže osnovni deo varijacija u stopama neizmirenja sa sistemskom komponentom. Ova intuicija se može uočiti sa Ilustracije 1, koja ilustruje kretanje stopa neizmirenja sa drugom najmanjom i drugom najvećom prosečnom vrednosti (stambeni krediti, i krediti dati malim i srednjim pravnim licima, redom), u odnosu na agregatnu stopu neizmirenja.

Tabela 1: *Deskriptivne statistike stopa neizmirenja po tipu proizvoda*

Redni broj	Tip	Srednja vrednost	Standardna devijacija	Minimum	Maksimum
1	Kreditne kartice	2,54	0,74	1,61	4,02
2	Gotovinski i potrošački krediti	2,93	0,65	2,07	4,10
3	Preduzetnici	3,39	1,24	1,45	5,91
4	Velika pravna lica	4,70	2,92	0,78	11,07
5	Lokalna samouprava	0,31	0,39	0,00	0,96
6	Stambeni krediti	1,89	0,57	0,83	2,65
7	Prekoračenja po tekućem računu	2,18	0,61	1,61	3,57
8	Poljoprivredna gazdinstva	2,13	0,57	1,47	3,16
9	Mikro krediti	8,46	6,54	1,21	20,30
10	Mala i srednja pravna lica	5,95	3,79	0,55	12,81
	Ukupno	**4,07**	**2,07**	**1,21**	**7,67**

Izvor: Baza podataka o neizmirenjima UBS

Prema načinu konstrukcije kategorija kreditnih plasmana moguća je situacija u kojoj se ista fizička lica pojavljuju u različitim kategorijama neizmirenja. Na primer, isto fizičko lice može kontribuirati neizmirenjima po osnovu kreditnih kartica, gotovinskih i potrošačkih kredita ili prekoračenja po tekućem računu. Međutim, ovde je važno dati nekoliko napomena. Prvo, indeks kreditne sposobnosti, definisan je jednakošću (1) na nivou kategorije, a ne na nivou dužnika. Ovakav pristup je u duhu Međunarodnog standarda finansijskog izveštavanja 9, koji podrazumeva da se ocena kreditne sposobnosti vrši po instrumentu, a ne po dužniku. Drugo, čak i kada se ocene kreditne sposobnosti vrše po licu umesto po instrumentu, veoma često i takve ocene ispoljavaju velike varijacije u zavisnosti od banke (Firestone & Rezende, 2016). Treće, autor nije posedovao mikro-podatke na nivou pojedinačnih dužnika, pa čak ni pojedinačnih banaka kako bi mogao da kontroliše fiksne efekte na ovim nivoima. U svakom slučaju, ovako organizovani podaci su korišćeni samo u svrhu ilustracije izloženog modela. Navedeni problem se svakako ne ispoljava kod plasmana preduzećima pošto tu ne dolazi do preklapanja korisnika kredita.



Ilustracija 1: Kretanje stopa neizmirenja sa drugim najmanjim i drugim najvećim prosekom u odnosu na agregatnu stopu neizmirenja

Izvor: Baza podataka o neizmirenjima UBS

Rezultati

Model ćemo ilustrovati na panel podacima opisanim u prethodnom odeljku. Prethodno ćemo ga postaviti u ekonometrijski pogodniji oblik. Kombinovanjem jednačina (1) i (2) dobijamo:

$$y_{it} = \bar{\alpha} + \bar{\beta}x_t + u_i + v_i x_t + \varepsilon_{it}. \tag{8}$$

U ovakvom obliku model je ekvivalentan mešovitom linearnom modelu (engl. *mixed-effects model*) sa jednim nivoom podele (po tipu kredita i). Model ima stohastički odsečak i nagib. Uzima u obzir fiksne efekte zajedničke za sve tipove kredita (sabirak $\bar{\beta}x_t$), slučajne efekte specifične za pojedinačne tipove kredita (sabirak u_i), kao i mešoviti uticaj fiksnih i slučajnih efekata (sabirak $v_i x_t$). Vrednosti y_{it} i x_t izračunate su na osnovu relacija (6) i (7).

Parametri modela ocenjeni su metodom maksimalne verodostojnosti pomoću algoritma maksimizacije očekivanja (engl. *expectation-maximization*, skraćeno EM).[2] Rezultati su sumirani u Tabeli 2. Vidimo da su svi parametri osim srednje vrednosti odsečka $\bar{\alpha}$ značajni na nivou od 0.05. Sam regresioni model je visoko značajan, što vidimo iz relativno velike vrednosti Wald statistike. Ocene parametara korelacione matrice slučajnih efekata (σ_u, σ_v i ρ), kao i standardne devijacije grešaka (σ_ε) su takođe visoko značajne, što vidimo iz njihovih intervala poverenja koji su svi sa iste strane nule. Relativno visoka vrednost koeficijenta korelacije ne predstavlja problem, pošto interval poverenja ne obuhvata jedinicu pa samim tim ni ne ukazuje na postojanje savršene korelisanosti između čistih slučajnih efekata, u_i, i njihovog doprinosa mešovitim efektima, v_i.

Poslednji red u Tabeli 2 prikazuje rezultate testa odnosa verodostojnosti (engl. *likelihood ratio*, skraćeno LR), kojim poredimo mešoviti linearni model sa njegovom restrikcijom na standardni regresioni model bez uticaja slučajnih efekata ($u_i \equiv 0$ i $v_i \equiv 0$). LR statistika je visoko značajna, posebno kada imamo u vidu da u EM pristupu predstavlja konzervativnu ocenu (Gutierrez *et al.*, 2001; McLachlan & Basford, 1988; Self & Liang, 1987; Stram & Lee, 1994). U ovo se možemo jednostavno uveriti i na osnovu Ilustracije 2, koja daje vizuelni prikaz zavisnosti latentne determinante kreditnog rizika po tipu kredita y_{it} od sistemskog faktora x_t. Pojedinačne linije povezuju tačke koje odgovaraju istim tipovima kredita i, što opravdava izvornu intuiciju upotrebe različitih koeficijenata pravca β_i uz sistemsku komponentu, kao i odsečaka α_i. Numeričke oznake na slici odgovaraju oznakama tipova kredita i iz Tabele 1.

[2] Algoritam su među prvima formalizovali Dempster *et al.* (1977). Za nešto moderniji prikaz algoritma videti reference Gupta & Chen (2010).

Tabela 2: *Ocene parametara mešovitog linearnog modela*

Parametar	Vrednost	St, greška	*p*-vrednost	Interval poverenja od 95%
$\bar{\alpha}$	–0,673	0,344	0,051	[–1,348, 0,002]
$\bar{\beta}$	0,722	0,152	0,000	[0,425, 1,019]
Wald (χ_1^2)	22,67		0,000	
σ_u	1,046	0,254		[0,650, 1,684]
σ_v	0,449	0,114		[0,273, 0,738]
ρ	0,962	0,027		[0,849, 0,991]
σ_ε	0,208	0,010		[0,190, 0,229]
LR (χ_3^2)	243,27		0,000	

Izvor: Proračuni autora

Vrednosti zavisne promenljive y_{it} na Ilustraciji 2 su negativne, što je u skladu sa definicijom u jednakosti (6) i odgovara levom kraju normalne funkcije raspodele. Naime, kako se sve stope neizmirenja kreću u intervalu od 0 do 20,3 procenta (Tabela 1), odgovarajuće vrednosti latentne promenljive y_{it} biće manje od $G(0.203) = -0.83$. Slično važi i za nezavisnu promenljivu x_t, koja je vezana za agregatnu stopu neizmirenja preko jednakosti (7). Ova stopa se kreće u intervalu od 1,21 do 7,67 procenata, pa će odgovarajuće vrednosti x_t biti u intervalu od –2,25 do –1,43.

Na osnovu ocena parametara modela iz Tabele 2 i jednačine (8), možemo izvesti regresionu predikciju očekivanih vrednosti zavisne promenljive:

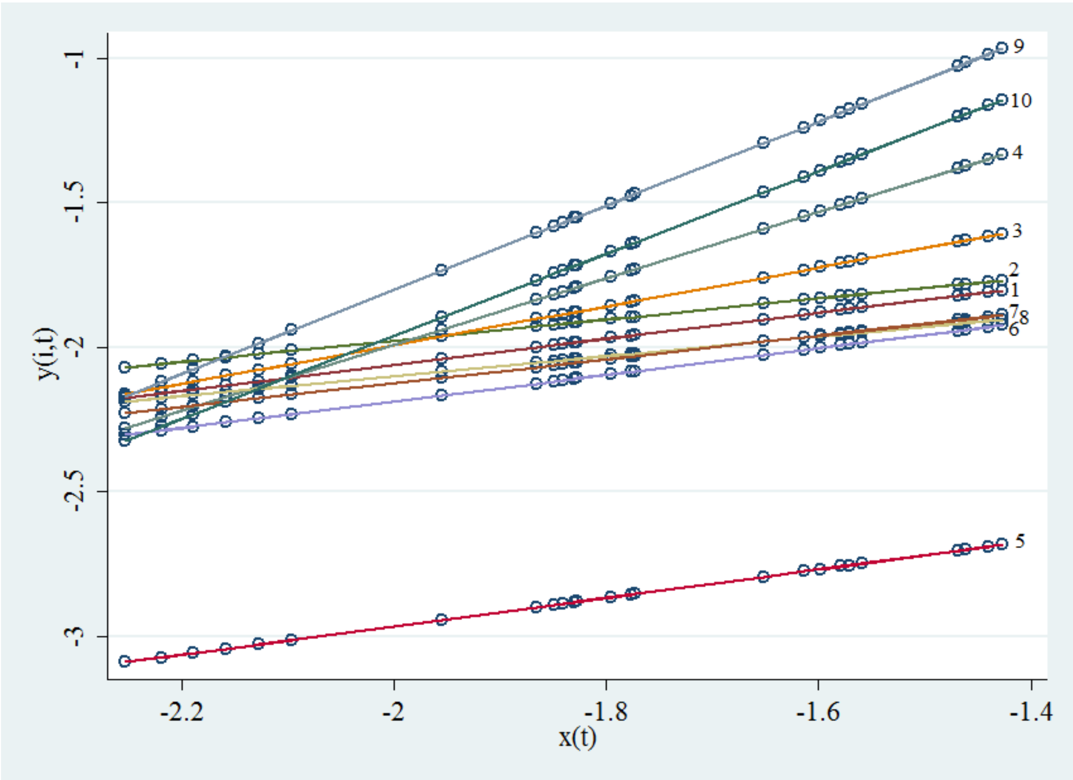
$$\hat{y}_{it} \equiv \mathbb{E}(y_{it}|x_t) = \hat{\bar{\alpha}} + \hat{\bar{\beta}}x_t + \hat{u}_i + \hat{v}_i x_t, \tag{9}$$

gde su $\hat{\bar{\alpha}}$ i $\hat{\bar{\beta}}$ tačkaste ocene parametara $\bar{\alpha}$ i $\bar{\beta}$ iz Tabele 2, a $\hat{u}_i$ i $\hat{v}_i$ su regresiona predviđanja slučajnih efekata za svaki od tipova kredita. Iz jednačina (9) i (6) možemo oceniti stope neizmirenja na osnovu mešovitog linearnog modela kao

$$\hat{d}_{it} = \Phi(\hat{y}_{it}), \tag{10}$$

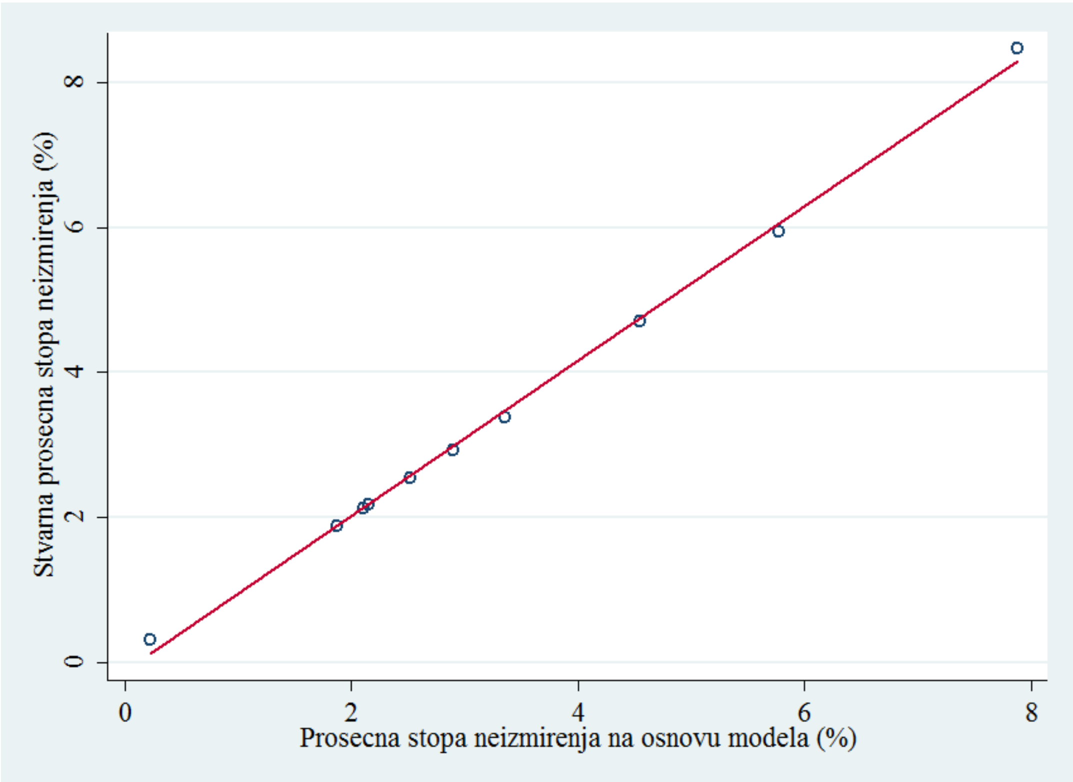
gde je $\Phi: \mathbb{R} \to [0,1]$ jednodimenziona funkcija normalne raspodele.

Ilustracija 3 poredi stvarne i modelske stope neizmirenja uprosečene po vremenu. Svaki od deset plavih kružića ima koordinatu $\left(\overline{\hat{d}_{i\cdot}}, \overline{d_{i\cdot}}\right)$, gde nadvučena linija označava usrednjavanje po vremenskoj dimenziji za svaki tip kredita i. Drugim rečima, nezavisna promenljiva $\overline{\hat{d}_{i\cdot}}$ na Ilustraciji 3 je vremenska srednja vrednost stope neizmirenja za svaki tip kredita i, dobijena usrednjavanjem vrednosti izračunate pomoću jednakosti (10). Zavisna promenljiva $\overline{d_{i\cdot}}$ predstavlja stvarne prosečne vrednosti stopa neizmirenja po tipu kredita i odgovara koloni sa oznakom 'Srednja vrednost' u Tabeli 1. Ove stvarne prosečne vrednosti kreću se u intervalu od 0,31 do 8,46 procenata. Crvena linija na Ilustraciji 3 predstavlja običan linearni fit i data je samo da bi olakšala vizuelizaciju. Lako je uočiti da model veoma dobro opisuje varijacije u prosečnim stopama neizmirenja po tipovima kredita.



Ilustracija 2: Zavisnost latentnih determinanti kreditnog rizika po tipu kredita y_{it} od sistemskog faktora x_t. Numeričke oznake odgovaraju tipovima kredita u Tabeli 1

Izvor: Proračuni autora



Ilustracija 3: Poređenje stvarnih i modelskih prosečnih stopa neizmirenja za deset tipova kredita

Izvor: Proračuni autora

Zaključak

U radu je predložen relativno jednostavan i praktično primenljiv model za razdvajanje determinanti rizika u kreditnim portfolijima poslovnih banaka na komponente. Model predstavlja linearnu vezu između latentne determinante kreditnog rizika pojedinačnog segmenta portfolija i sistemskog faktora. Ove determinante su, redom, povezane sa stopama neizmirenja unutar svakog segmenta i agregatnom stopom neizmirenja svih datih kredita. Varijacije u uticaju sistemske komponente na pojedinačne segmente se stoga svodi na varijacije u koeficijentima uz sistemski faktor. Ulogu idiosinkratske komponente igra odsečak koji takođe varira po segmentima.

Model je ilustrovan na podacima Udruženja banaka Srbije o plasmanima u statusu neizmirenja. Korišćen je panel od deset portfolio segmenata, organizovanih na osnovu tipova kredita, u periodu između četvrtog kvartala 2012. i četvrtog kvartala 2018. godine. Parametri modela ocenjeni su pomoću metoda mešovitih linearnih modela sa jednim nivoom podele, algoritmom maksimizacije očekivanja. Model pokazuje visoku moć statističkog objašnjavanja podataka unutar uzorka. Dobijeni rezultati ukazuju da predložena dekompozicija veoma dobro opisuje dinamiku stopa neizmirenja po tipovima kredita, kao i razlike u prosečnim stopama neizmirenja po tipovima kredita.

Agregatna stopa neizmirenja je algebarskom transformacijom povezana sa jedinstvenim faktorom, koji u mešovitom modelu predstavlja fiksne efekte zajedničke za sve segmente. Ona stoga igra ulogu jedinstvene sistemske komponente koja jednostavno oponaša uticaje stvarnih makroekonomskih faktora rizika, a ne iziskuje eksplicitno modeliranje ove veze. Ova osobina ima potencijalno veliki praktični značaj, posebno u uslovima nedovoljno dugačkih vremenskih serija gde je teško ustanoviti robusnu statističku vezu. Pored toga, model omogućava da se bilo kakav pristup „gledanja unapred" (engl. *forward looking*) svede na modeliranje dinamike samo jedne—agregatne—stope neizmirenja, umesto svakog segmenta ponaosob. Uz jednostavnost pristupa, to čini osnovne doprinose ovog rada.

Važno je napomenuti da je zbog ograničenja u broju raspoloživih podataka i varijacija po obe dimenzije panela model testiran samo unutar uzorka. Bilo bi svakako ne samo interesantno nego i neophodno uveriti se u njegove performanse i van uzorka. Na primer, sa dodatnim raspoloživim podacima o agregatnim stopama neizmirenja (ili njihovim projekcijama dobijenim nekim nezavisnim metodom) moguće je uporediti predviđene i realizovane stope neizmirenja po tipovima kredita. Takođe, bilo bi interesantno posmatrati i podatke na nivou pojedinačnih kredita, te proveriti da li se na njima može uočiti dodatni faktor koji bi opisao specifično ponašanje pojedinačnih dužnika. Mešoviti linearni model je upravo pogodan za ovakvu analizu, jer omogućava dodatne nivoe podele na „ugnježdene" potkategorije. Ovim bi se napravila dobra paralela sa delom literature koji sugeriše razdvajanje na sistemsku, sektorsku i „pravu" idiosinkratsku komponentu kreditnog rizika.

Napomena

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ONE-FACTOR MODEL FOR DEFAULT RATES BY LOAN TYPE

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Summary:

This paper investigates the link between default rates by loan types and the systemic credit risk component. This link is described by a linear model that combines systemic and idiosyncratic contributions. The systemic component is a latent factor that depends directly on the aggregate loan default rate, while the idiosyncratic component drives specific variations of default rates across loan types. By transforming observable risk measures, the model can be econometrically represented as a mixed-effects model, where the systemic and idiosyncratic components represent, respectively, the slope and the intercept that are specific for each loan type individually. The proposed model is illustrated on a panel of defaulted loans of the Association of Serbian Banks. The obtained results show the model's very high power in explaining average default rates for all loan types. Thus, the aggregate default rate plays the role of a unique systemic component that mimics the influence of fundamental macroeconomic risk factors easily, without the necessity to model this relationship explicitly.

Keywords: credit risk; default rate; systemic factors; mixed-effects model

JEL classification: G21, C23.

Introduction

Decomposing risk into factors that describe its various sources is one of the fundamental issues in financial economics. The contributions of individual risk components are significant because they enable monitoring sources of risk that can lead to portfolio losses. Identifying sources of risk is particularly important for complex portfolios of financial institutions and certain types of instruments where factors are not manifestly identifiable, such as some types of derivatives.

However, the standard division of risk components into systemic and idiosyncratic (Bodie *et al.*, 2021) implicitly subsumes the price risks of instruments in the securities markets. Relatively less attention has so far been paid to the analogous decomposition of credit risk, which is undoubtedly the most critical source of risk for commercial banks and other credit institutions. This approach has significantly changed after the Global Financial Crisis, which has unequivocally shown that we cannot consider debt-specific factors alone in managing the credit risk of a portfolio. It is necessary to consider the possible spillover effects of individual debtors on entire industries and even on the economy as a whole (Ballester *et al.*, 2016). We can understand these spillovers (at least on an abstract level) precisely through the systemic component of credit risk, which is common to all types of debtors and thus to entire portfolios of banks.

The systemic component originates from macroeconomic risk factors and, in that respect, is quite analogous to market risk. For financial stability, which has become one of the central issues in post-crisis regulatory frameworks (BCBS, 2011), the impact of systemic factors on financial and sovereign sectors are of particular importance (Acharya *et al.*, 2014), as well as the channel of spillovers from these to non-financial sectors (Gross & Siklos, 2020). However, at the practical level, in the Basel regulatory framework, these factors are generally not modeled explicitly within Pillar I.[1] Instead, they are managed indirectly through countercyclical capital buffers and concentration risk. Repullo & Saurina (2011) criticized the mechanical application of countercyclical buffers and argued that it enhances the inherent procyclicality of regulatory capital requirements.

Credit risk management strategies of financial institutions must consider business cycles and related variations in the quality and structure of their loan portfolios (Jiménez & Saurina, 2006). For these strategies to be sustainable in the long run, banks have to monitor how sensitive their portfolios are to changes in various macroeconomic and financial indicators. Therefore, the degree to which specific economy segments contribute to systemic risk becomes among the crucial ones (Novales & Chamizo, 2019). To some extent, we can equate it with the exposure of certain parts of a financial institution's loan portfolio to the systemic risk component.

The literature on the topic of credit risk decomposition into factors is mostly of more recent date. Rosen & Saunders (2010) pointed out that one of the primary challenges related to this topic is the nonlinear influence of factors on observable quantities. They proposed a multifactor model linked by the Hoeffding decomposition to total portfolio losses. The variables they used were a combination of macroeconomic, geographical, sectoral and market factors (interest rates, exchange rates, stock volatility). Chamizo & Novales (2016) used credit derivatives to separate the impact of systemic from the idiosyncratic component of large European companies' credit risk using market data on CDS spreads. Earlier, the determinants of CDS spreads were studied by Ericsson *et al.* (2009). Using a principal com-

[1] The exception is the Internal Ratings Based (IRB) approach, where the systemic component is indirectly modeled by "portfolio risk". This factor in the IRB approach always takes a fixed value in the capital requirements formula. It is obtained using a one-factor Gaussian copula model (Vasiček, 2002)

ponent analysis, they were able to identify one dominant common factor for all credit exposures in the sample. The residual contribution of non-systemic components can be most likely attributed to a combination of sector and individual-specific risk factors (Bhansali *et al.*, 2008; Novales & Chamizo, 2019).

The related issue of counterparty risk decomposition has been addressed by de Graaf *et al.* (2018). They additionally pointed out the importance of decomposing the components into as few factors as possible due to more straightforward monitoring and interpretation. Campbell *et al.* (2001) studied the dynamics of the systemic component of market risk and showed that it has the behavior of a "leading indicator" to the idiosyncratic component. In the context of credit risk, the existing literature assesses the dynamics of the systemic component mostly from bond and credit derivative market data. Alternatively, it is directly related to macroeconomic fundamentals. In academic research, relatively little attention has so far been paid to the decomposition of risk factors of commercial banks' loan portfolios and other illiquid financial instruments.

By recognizing the apparent research gap in the literature, this paper aims to propose an implementable and straightforward model for the decomposition of credit risk determinants. Following the general logic of factor models used in asset pricing, we have developed a linear model with one systemic risk factor. Although this factor is latent in the model, it is related by a simple algebraic transformation to the observable default rate of all loans extended by the Serbian banks. The residual contribution to the total risk of an instrument or a portfolio is attributed to the loan type-specific risk component. An illustration of the model is given using the panel data of the Association of Serbian Banks (ASB) on defaulted loans.

The remainder of the paper is organized as follows. The next section presents a theoretical model for credit risk decomposition. The third section describes the data used to illustrate the model, shown in the fourth section. The last section contains concluding remarks.

Model

We propose a model that separates the credit risk of individual groups of loans (i.e., loan types) into contributions from systemic and type-specific risk components. The logic of the model is somewhat similar to the factor models used in asset pricing. In particular, we will assume that a representative debtor's creditworthiness within each group i can be described by the latent factor y. The following linear relationship gives the dynamics of changes in this factor:

$$y_{it} = \alpha_i + \beta_i x_t + \varepsilon_{it}. \tag{1}$$

In equation (1), y_{it} is the creditworthiness index of the loans of type i at time t, x_t is the latent creditworthiness index of all debtors (i.e., all loan types considered as an aggregate), while ε_{it} represents a random deviation from the linear relationship. The parameters α_i and β_i are specific for each loan type and measure the idiosyncratic and specific components in each group, respectively. Geometrically, we can interpret them as individual slopes and intercepts in regression by groups. More specifically, we will assume they can be represented as deviations from their means:

$$\alpha_i = \bar{\alpha} + u_i$$
$$\beta_i = \bar{\beta} + v_i. \tag{2}$$

The deviations u_i and v_i are individually independent and identically distributed. However, we allow for the presence of correlations between the two. Asymptotically, we can regard them as elements of a two-dimensional random vector from a joint normal distribution:

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim \mathbf{\Phi}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}\right), \tag{3}$$

where $\mathbf{\Phi}: \mathbb{R}^2 \to [0, 1]$ is the normal distribution function described by the vector of means and the covariance matrix.

Since y_{it} and x_t are latent factors, we have to relate them to observable quantities. Specifically, we will assume that they are related to the corresponding default rates in the following way:

$$d_{it} = \mathbb{P}(y_{it} < \mathbb{E}(y_{it}|x_t)|x_t)$$
$$D_t = \mathbb{P}(x_t < \mathbb{E}(x_t)), \tag{4}$$

where $\mathbb{P}$ represents the probability measure. We measure the default frequency for loans of type i at time t with the corresponding default rate d_{it}, while the default frequency for all the loans at time t is measured by the aggregate default rate D_t. Expressions in (4) establish the equality between (conditional) probabilities that the latent factors are below their expected values and default rates as the sample analogs of these probabilities. Using the standard assumption of linear models that the error terms ε_{it} are independent and identically distributed random variables with mean zero and variance σ_ε^2, orthogonal on x_t, it is easy to show that the quantity

$$y_{it} - \mathbb{E}(y_{it}|x_t) = u_i + v_i x_t + \varepsilon_{it} \tag{5}$$

is asymptotically normally distributed. This property implies that we can relate it to the data using

$$y_{it} = G(d_{it}), \tag{6}$$

where $G: [0, 1] \to \mathbb{R}$ is the inverse of a univariate normal distribution function. By analogy, we will use the same relationship for the aggregate default rates:

$$x_t = G(D_t). \tag{7}$$

Data

To illustrate the model, we will use aggregated anonymized data from the ASB on the number and the monetary value of loans in default status. The data are granulated by loan type, so there is a total of ten different types. The data frequency is quarterly and covers the period between the last quarter of 2012 and the last quarter of 2018. We define the default rate as the fraction of the total outstanding monetary amount of defaulted loans in the total outstanding amount of all extended loans. The rationale behind this choice is explained in more detail in some previous works (see, for example, Friedson, 1991 or Božović, 2019).

Table 1 shows the summary statistics for the data by loan type. The division of loans by types was performed following the one used in the ASB database. The last row of the table (marked "Total") shows the data for all banks in the sample, i.e., data obtained based on aggregate default rates. We can easily see one regularity from the table: higher average default rates are paired with higher standard deviations of these rates. The intuition for this phenomenon is similar to that of market instruments, where a higher average rate of return is coupled with a higher risk measured by return volatility. Of course, this analogy should not be taken literally. The basic idea of the model we present in this paper relates to the principal source of the variations in default rates to the systemic component. This intuition can be seen in Figure 1, which illustrates the evolution of default rates having the second-lowest and the second-highest average values (housing loans and loans to medium and small enterprises, respectively), relative to the aggregate default rate.

Table 1: *Summary Statistics of Default Rates by Loan Type*

No.	Loan type	Mean	Standard deviation	Minimum	Maximum
1	Credit cards	2.54	0.74	1.61	4.02
2	Cash and consumer loans	2.93	0.65	2.07	4.10
3	Loans to entrepreneurs	3.39	1.24	1.45	5.91
4	Loans to large corporate entities	4.70	2.92	0.78	11.07
5	Loans to local governments	0.31	0.39	0.00	0.96
6	Mortgage loans	1.89	0.57	0.83	2.65
7	Overdrafts	2.18	0.61	1.61	3.57
8	Agricultural loans	2.13	0.57	1.47	3.16
9	Micro loans	8.46	6.54	1.21	20.30
10	Loans to small and medium enterprises	5.95	3.79	0.55	12.81
	Total	**4.07**	**2.07**	**1.21**	**7.67**

Source: Loan default database of ABS

Based on how the loan types are constructed, multiple individuals may appear in different loan default categories. For example, the same individual can contribute to defaults on credit cards, cash and consumer loans, or overdrafts. However, it is essential to give a few remarks at this point. First, the creditworthiness index is defined by equation (1) on the loan type level, not on the individual level. This approach follows the spirit of the International Financial Reporting Standard 9, which requires that the creditworthiness should be estimated by instrument rather than the borrower. Second, even if the creditworthiness was estimated on the borrower level instead of the instrument level, it is expected that such estimates exhibit significant variations across banks (Firestone & Rezende, 2016). Third, the author did not have any micro data on an individual- or bank-level to control for the relevant fixed effects. In any case, the database was used for model illustration only. The issue is not relevant for loans extended to companies since there is no overlap between borrowers across different loan types.

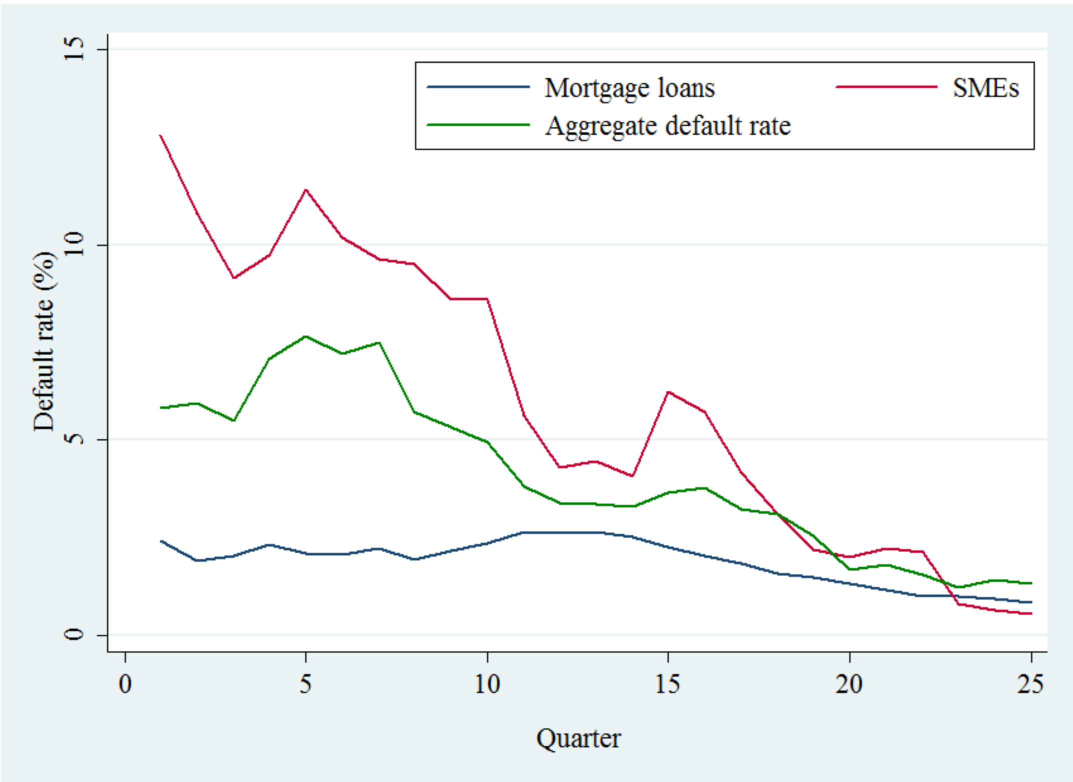

Figure 1: Evolution of Default Rates with the Second-Lowest and the Second-Highest Average, Compared to the Aggregate Default Rate

Source: Loan default database of ABS

Results

We will illustrate the model on the panel data described in the previous section. Before doing so, we will cast it in a more convenient form from the econometric point of view. Combining equations (1) and (2), we get:

$$y_{it} = \bar{\alpha} + \bar{\beta}x_t + u_i + v_i x_t + \varepsilon_{it}. \tag{8}$$

In this form, the model is equivalent to a single-level mixed-effects model. The level here refers to the loan type i. The model has a random intercept and slope. It takes into account the fixed effects common to all loan types (through $\bar{\beta}x_t$), loan type-specific random effects (through u_i), and the mixed effect of fixed and random effects (through $v_i x_t$). The values for y_{it} and x_t are calculated using equations (6) and (7).

The parameters of the model were estimated by the maximum likelihood method using the expectation-maximization (EM) algorithm.[2] The results are summarized in Table 2. We see that all parameters except the mean value of the intercept $\bar{\alpha}$ are significant at the 0.05 level. The regression model itself is highly significant, as we see from the relatively large value of Wald statistics. The estimates of the parameters of the correlation matrix of random effects (σ_u, σ_v and ρ), as well as the standard deviations of the error term (σ_ε) are also highly significant, as their confidence intervals are all on the same side of zero. The relatively high value of the correlation coefficient is not an issue, since the value $\rho = 1$ is outside the confidence interval. Thus, at a 95-percent confidence level, there is no perfect correlation between pure random effects, u_i, and their contribution to mixed effects, v_i.

The last row in Table 2 shows the likelihood ratio (LR) test results, which compares the mixed linear model with its restriction to the standard regression model without the influence of random effects ($u_i \equiv 0$ i $v_i \equiv 0$). The LR statistics are highly significant, especially when considering that it represents a conservative assessment in the EM approach (Gutierrez *et al.*, 2001; McLachlan & Basford, 1988; Self & Liang, 1987; Stram & Lee, 1994). We can easily verify this from Figure 2, which gives a visual representation of the dependence of the latent loan type-specific credit risk determinant y_{it} on the systemic factor x_t. The individual lines connect the points corresponding to the same types of loans. This visualization justifies the original intuition of using different slope coefficients β_i corresponding to the systemic component and the different intercepts α_i. The numerical labels in the figure correspond to the designations of loan types from Table 1.

[2] Dempster *et al.* (1977) were among the first to describe this algorithm formally. For a more recent presentation of the algorithm, see Gupta & Chen (2010).

Table 2: *Parameter Estimates in the Mixed-Effects Model*

Parameter	Value	St. error	*p*-value	95% confidence interval
$\bar{\alpha}$	−0.673	0.344	0.051	[−1.348, 0.002]
$\bar{\beta}$	0.722	0.152	0.000	[0.425, 1.019]
Wald (χ^2_1)	22.67		0.000	
σ_u	1.046	0.254		[0.650, 1.684]
σ_v	0.449	0.114		[0.273, 0.738]
ρ	0.962	0.027		[0.849, 0.991]
σ_ε	0.208	0.010		[0.190, 0.229]
LR (χ^2_3)	243.27		0.000	

Source: Author's calculations

In Figure 2, all values of the dependent variable y_{it} are negative, which is a consequence of the definition given by equation (6) and corresponds to the left tail of the normal distribution function. Namely, all default rates take the values between 0 and 20.3 percent (Table 1). Thus, the corresponding values of the latent variable y_{it} will all be below $G(0.203) = -0.83$. Similarly, the independent variable x_t is related to the aggregate default rate through equation (7). This rate takes values in the interval between 1.21 and 7.67 percent, implying that the corresponding values of x_t will be between –2.25 and –1.43.

Based on the estimates of the model parameters from Table 2 and equation (8), we can derive the regressional prediction of the expected values of the dependent variable:

$$\hat{y}_{it} \equiv \mathbb{E}(y_{it}|x_t) = \hat{\bar{\alpha}} + \hat{\bar{\beta}}x_t + \hat{u}_i + \hat{v}_i x_t, \tag{9}$$

where $\hat{\bar{\alpha}}$ and $\hat{\bar{\beta}}$ represent the point estimates for $\bar{\alpha}$ and $\bar{\beta}$ in Table 2, while $\hat{u}_i$ and $\hat{v}_i$ are the regressional predictions of random effects for each loan type. From equations (9) and (6), we can estimate the default rates implied by the mixed-effects model as:

$$\hat{d}_{it} = \Phi(\hat{y}_{it}), \tag{10}$$

where $\Phi: \mathbb{R} \to [0,1]$ is the univariate normal distribution function.

Figure 3 compares actual and model-implied time-averaged default rates. Each blue circle has a coordinate $\left(\overline{\hat{d}_{i\cdot}}, \overline{d_{i\cdot}}\right)$, where the bar indicates averaging over the time dimension for each loan type i. In other words, the independent variable $\overline{\hat{d}_{i\cdot}}$ in Figure 3 is a time-averaged default rate for each loan type i, obtained by averaging the model-implied values from equation (10). The dependent variable $\overline{d_{i\cdot}}$ represents the actual average default rate by loan type and corresponds to the column labeled 'Mean' in Table 1. These actual average values range between 0.31 and 8.46 percent. The red line in Figure 3 represents a simple linear fit and is given only to facilitate visualization. It is easy to see that the model describes the variations in average default rates by loan type quite well.

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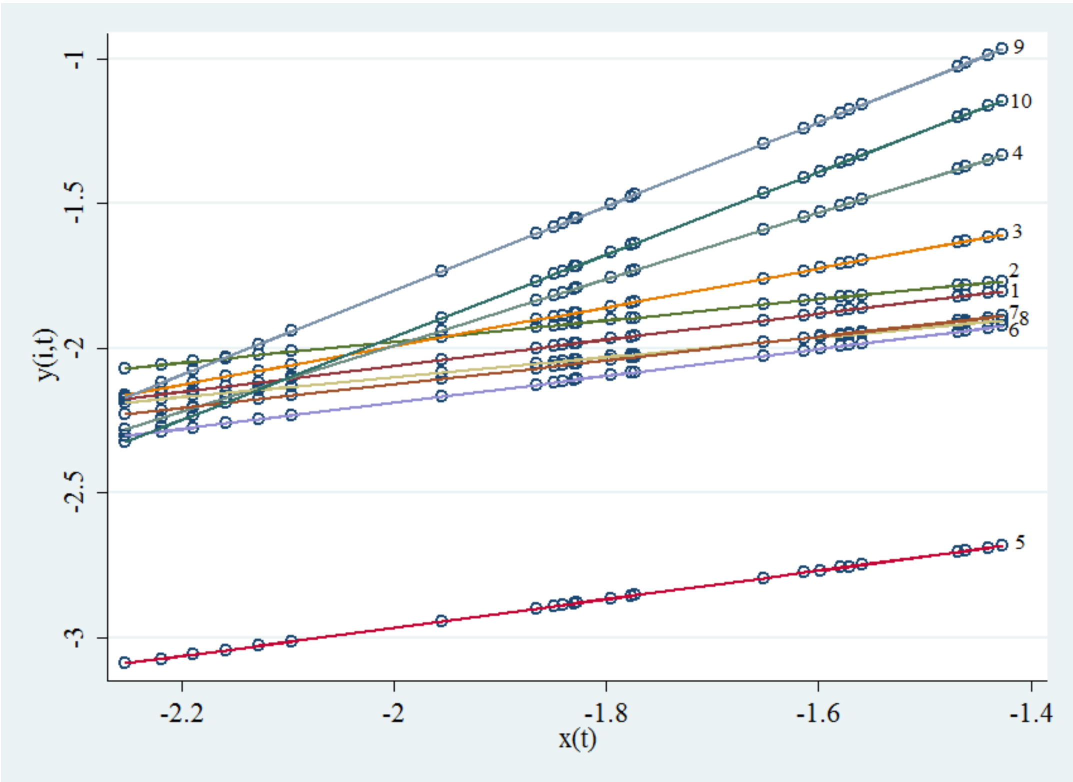

Figure 2: Dependence Between the Latent Credit Risk Determinant y_{it} and the Systemic Factor x_t. Numerical labels correspond to the loan types from Table 1

Source: Author's calculations

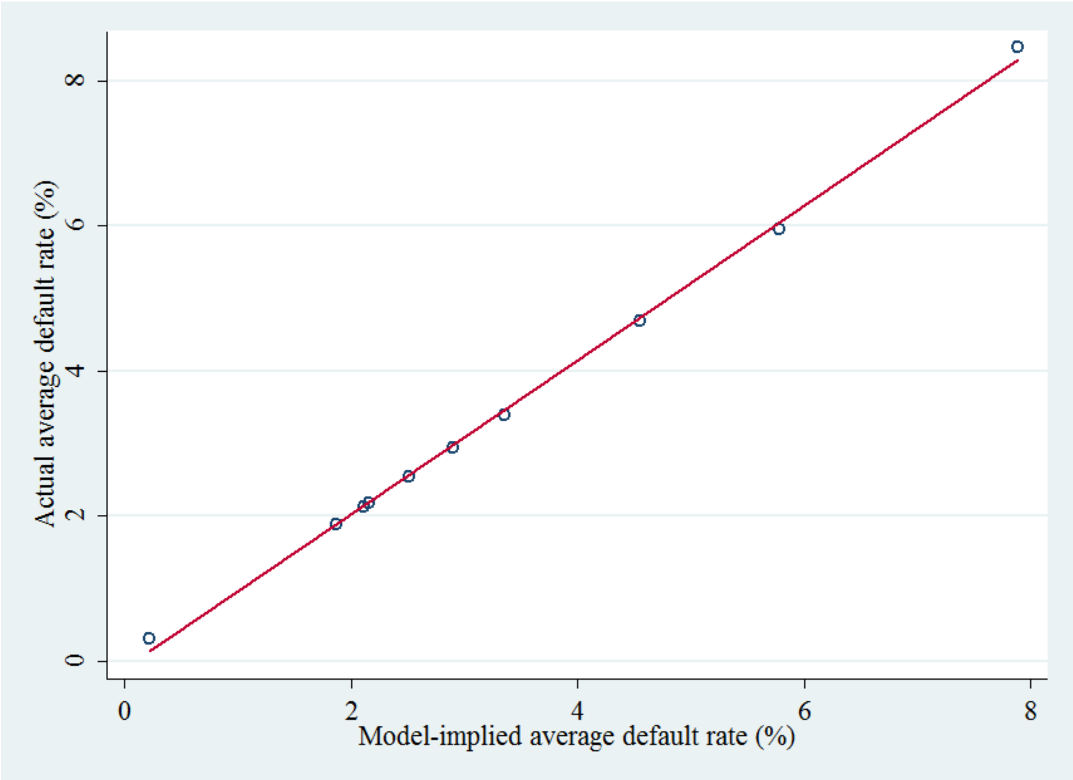

Figure 3: Actual vs. Model-Implied Average Default Rates for Ten Loan Types

Source: Author's calculations

Conclusion

The paper proposes a relatively simple and practically applicable model for separating risk determinants in commercial banks' loan portfolios into components. The model represents a linear relationship between the latent determinant of the credit risk of an individual portfolio segment and the systemic factor. These determinants are, in turn, related to the default rates within each segment and the aggregate default rate of all loans extended. Variations in the influence of the systemic component on individual segments are reduced to variations in the coefficients associated with the systemic factor. The role of the idiosyncratic component is played by the intercept that also varies by segments.

The model is illustrated on the data of the Association of Serbian Banks on defaulted loans. We used a panel of ten portfolio segments, organized based on loan types, between the fourth quarter of 2012 and the fourth quarter of 2018. The model shows high explanatory power for data within the sample. The obtained results indicate that the proposed decomposition well describes the dynamics of default rates by loan type and the differences in average default rates by loan type.

An algebraic transformation relates the aggregate default rate to a unique factor, representing fixed effects common to all segments in a mixed-effects model. Therefore, it plays the role of a unique systemic component that mimics the influences of real macroeconomic risk factors, rather than requiring explicit modeling of this relationship. This feature has potentially great practical significance, especially in insufficiently long time series where it is difficult to establish a robust statistical relationship. The model also allows any forward-looking approach to be reduced to modeling the dynamics of only one— aggregate—default rate instead of modeling the rates for each segment individually. Along with the simplicity of the approach, this makes the main contribution of this paper.

It is important to note that the model was tested only within the sample due to limitations in the number of available data and variations across both panel dimensions. It would undoubtedly be both interesting and necessary to verify its performance out of the sample. For example, with additional available data on aggregate default rates (or their projections obtained by some independent method), it is possible to compare projected and realized default rates by loan type. It would also be interesting to use the data at the individual loan level and check whether we can identify an additional factor that would describe the specific behavior of individual debtors. The mixed-effects model is precisely suitable for this analysis because it allows additional layers of division into “nested” subcategories. This extension would draw a good parallel with the literature strand that suggests a separation of credit risk into systemic, sectoral, and “real” idiosyncratic components.

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