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YOUNG PUPILS' ABILITY TO SOLVE PERSPECTIVE-TAKING PROBLEMS[1]

Abstract: Spatial ability is an integral part of mathematics teaching and learning, but not every component of this ability has received enough research attention. In this paper, we focus on young pupils' ability to solve two types of imaginary perspective-taking (IPT) problems given in the form of a test (the paper–pencil test). The results show a difference in solving imaginary perspective-taking problems between preschoolers and second-grade pupils who took part in this research. Still, even the second-grade pupils have not fully developed this special spatial ability because they are slightly less successful in appearance IPT2 tasks than in visibility IPT1 tasks. We noticed individual differences in both age groups. In addition, the preschool sample from Serbia is equally successful as the children from the Netherlands and significantly better than the children from the Cyprus sample of the same age reported by Van den Heuvel-Panhuizen, Elia and Robitzsch (2015). The general conclusion and educational implication are that imaginary perspective-taking ability should be nurtured more in early school years.

Keywords: imaginary perspective-taking, visibility, appearance, spatial reasoning, young pupils, Serbia.

INTRODUCTION

Determining the position occupied by objects in space has an adaptive significance for all living beings. We look for a path to navigate, manipulate objects in our daily lives, or imagine situations in which we may find ourselves (Newcombe, Huttenlocher 1992; Sinclair, Bruce 2014). In addition, it is often useful to be able to predict the position that an object can occupy in space with respect to different viewpoints, i.e. perspective-taking (Van den Heuvel-Panhuizen, Elia, Robitzsch 2014, 2015). As spatial reasoning is a key cognitive ability illustrated by the above

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examples and supported by scientific research (Davis, Spatial Reasoning Study Group 2015), this article examines the ability of children of preschool and younger school age in Serbia to take an imaginary perspective.

Modern mathematics curricula emphasize the need to start with 3D geometry at an early age (Van den Heuvel-Panhuizen, Elia, Robitzsch 2014), which is in line with Freudenthal's (1973) position on early geometry learning. Therefore, spatial ability is important for young children's learning, and it is valuable to gain as much insight as possible into how children develop this ability. The major component of spatial ability is imagining objects from different observer perspectives (Van den Heuvel-Panhuizen, Elia, Robitzsch 2014, 2015). We focus on this specific spatial competence of preschoolers and children of younger school age, i.e. on the competence of imaginary perspective taking (IPT), which means children's ability to take a certain point of view mentally and to be able to make conclusions about the positions of objects from an imaginary perspective spatial competence (IPT).

THEORETICAL BACKGROUND

We follow a new path in mathematics education research. In a comprehensive review of recent research within the CERME 11 thematic working group (TWG) on the early years of mathematics, POEM4 conference and ICME-13 monograph *Contemporary research and perspectives on early childhood mathematics education* we have identified three recurring themes: 1) early interventions and their effects, 2) facilitating factors for learning and development and 3) key mathematical concepts that can be observed in children (Björklund, Van den Heuvel-Panhuizen, Kullberg 2020). We follow the directions of Björklund, Van den Heuvel-Panhuizen and Kullberg (2020) for research on early mathematics teaching and learning to get a deeper insight into what mathematics means to young children and how the foundations can be laid for the domains of spatial and geometric thinking.

In the early years, there is still an insufficient focus on the development of geometry and spatial thinking (Sinclair, Bruce 2014; Uttal, Cohen 2012; Đokić 2018; Đokić, Boričić, Jelić 2022). The first indicator that this situation is changing for the better is extensive recent research that consistently shows a strong connection between children's spatial abilities and achievements in mathematics and science (Newcombe 2010), which increases the probability of achieving better achievements in STEM disciplines (science, technology, engineering, and math) (Young, Levine, Mix 2018; Wai, Lubinski, Benbow 2009). Secondly, a growing number of indicators that children come to school with developed informal spatial reasoning exist (Bryant 2008; Ishikawa, Newcombe 2021), which is often not supported in the continuation of mathematics education through curricula that require the development of numerical and algebraic ways of thinking.

COMPONENTS OF SPATIAL ABILITY

Spatial ability is considered an autonomous intellectual ability (Clements, Battista 1992). Factor analysis identified specific abilities that make up spatial ability: spatial orientation, mental rotation, and spatial visualization (1925–1979) (Mix, Cheng 2012: 200). Each of the listed components refers to specific requirements. Spatial orientation refers to the perception of the position and changes of objects in space, mental rotation to the mental manipulation of remembered objects with regard to the degree of rotation, and spatial visualization to understanding complex spatial patterns and imagined movements of objects in space. In addition to the specified specific components, a meta-analysis of definitions of spatial abilities (1975–2011) also mentions disembedding, spatial perception, and imaginary perspective-taking (adapted from Uttal et al. 2013: 355). Imaginary perspective-taking is the visualizing of an environment in its entirety from a different position.

Isolating and researching each of the mentioned specific spatial abilities is important because the analyses of individual differences show that people who are good in one component are not necessarily good in others (Newcombe, Uttal, Sauter 2013). Here is an example of the research that points to a distinction between mental rotation and perspective-taking. Although formally and logically equivalent, mental rotation and perspective-taking are psychologically very different constructs. Subsequent studies have confirmed neural differences, but also individual differences that separate these two spatial abilities (Newcombe, Huttenlocher 2006). Newcombe and Huttenlocher state that Piaget conceptualizes them as a part of different developmental lines and that they occur at different ages in children. Thus, it is easier to select an image that shows the appearance of an object or sequence after imagining it rotating on its axis than after imagining moving to take a different perspective on the same object or sequence. However, mental rotation is not always simpler than perspective-taking. When people ask themselves which object in a sequence would be in a certain position relative to them after the transformation (e.g. *What would be closest? What would be to the left?*), imaginary perspective-taking is easier than mental rotation.

There is a fairly extensive literature showing the relationship between math performance and specific spatial abilities: mental rotations, spatial visualization, and visuospatial working memory (e.g. Lubinski 2010; Newcombe 2010; Uttal et al. 2013). There is little to no evidence for other specific spatial abilities, such as imaginary perspective-taking, map reading, or model use (Mix, Cheng 2012). The literature is also geared towards older children and adolescents, leaving us with relatively little information on the relationship between spatial abilities and math performance in younger children.

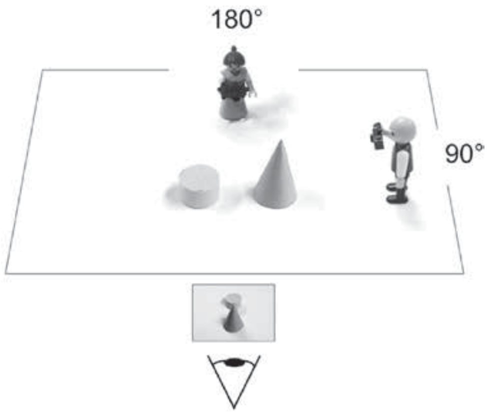
One of the few and earliest studies that singled out the imaginary perspective-taking as a separate, specific ability is the research of Guay and McDaniel (1977) who tested whether the ability to coordinate multiple points of view is related to achievement on the math subtest of the Iowa Test of Basic Skills. It is interesting that there was a significant correlation in the 5th, 6th, and 7th grades but not in the 2nd, 3rd, and 4th grades.

Although imaginary perspective-taking may not seem as obviously related to mathematics as other specific abilities, such as spatial visualization, there is a reason to suspect a connection. First, it seems likely that imaginary perspective-taking would be related to geometry, given the need to conceptualize the shapes of geometric objects from different angles and perspectives. A good perspective-taking ability may also reflect a level of "spatial flexibility" that would allow children to see equivalence in different situations, such as different solutions in algebra and proofs in geometry. Finally, the same processes that allow people to label and maintain separate viewpoints in working memory can also help them complete multi-step math problems, such as solving inequalities or adding fractions with different denominators. This is because keeping track of what is happening on both sides of the equal sign could involve the same processes needed to keep track of the multiple viewpoints (Mix, Cheng 2012; Uttal et al. 2013).

IMAGINARY PERSPECTIVE-TAKING

Emphasizing the importance of transdisciplinary research, Bruce et al. (2017) examined the lack of mutual understanding of the disciplines of mathematics education, psychology, mathematics, and neuroscience on the concept and development of spatial reasoning. To gain an insight into the existing connections between the disciplines, they conducted a network analysis that showed that taking a perspective is a representative case of overlapping psychological and mathematical education research. In psychology, imaginary perspective-taking is a cognitive construct that originated in the seminal work of Jean Piaget (Piaget, Inhelder 1948/1967). In Piaget's classic test, "The Three Mountains Task", a child is presented with a landscape scene and asked to describe it from other perspectives. The construct is often associated with egocentrism and consideration of others' points of view (Piaget 1932/1997). More recently, variations of the task have contributed to a better understanding of children's perspective-taking abilities. For example, Frick, Möhring and Newcombe (2014) designed a task involving a three-dimensional setting. Children were asked which of the two Playmobil figures took the photo shown above the picture of the eye (Figure 1). Their results show that egocentric errors occur mostly at the age of four and five and increase with the complexity of the request, but also decrease with age, i.e. the number of children who perform above the chance level increases at the age of five and six, while children at the age of seven and eight significantly reduce the number of errors, although even then individual differences are considerable.

Figure 1. Imaginary perspective-taking task (Frick, Möhring, Newcombe 2014: 3)



Earlier, Newcombe (1989) found that many studies rejected Piaget's age limitations, showing that children can overcome their egocentrism early in the preschool years. Thus, Newcombe and Huttenlocher (1992) provide evidence that even three-year-olds can solve perspective-taking problems by shifting from an egocentric to an allocentric frame of reference, i.e. children of preschool age can show the ability to take different perspectives of the object. Not all components of spatial abilities develop at the same age and particularly mental rotation appears a year or two later. However, variant forms remain severe at this age and are slow to develop, with good achievements not evident until the age of 10 (Newcombe, Huttenlocher 1992). Although imaginary perspective-taking is poorly researched within mathematics education (unlike psychological research), it becomes present in curricular outcomes, e.g. in three-dimensional structures. Bruce et al. (2017) report an outcome in the Ontario mathematics curriculum: build three-dimensional models using stacking cubes, according to a given isometric sketch or different views of the structures (top, side and front) (give an example problem: Given a top, side and front view of a complex structure, stack the body using the smallest possible number of cubes) (Ontario Ministry of Education 2005: 92). Although the given example is quite simple, it draws our attention to the presence of imaginary perspective-taking in many aspects of mathematics and in many activities such as drawing a structure, assembling and disassembling it, and navigation and mapping.

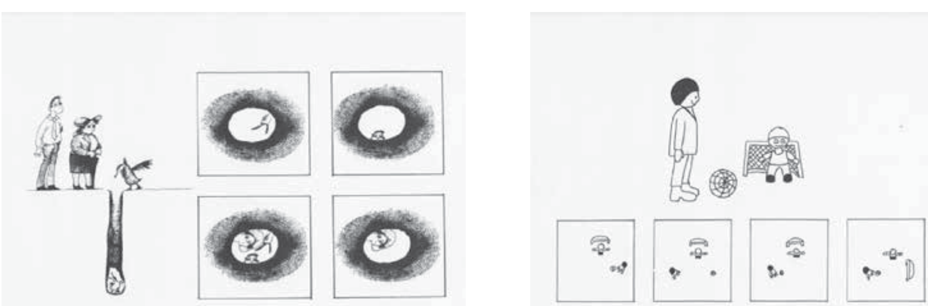
We know little about how spatial ability affects children's development. Does the relationship between spatial abilities and math performance change over time? Is the relationship more important at one point in development than at another? Does it involve children's specific abilities or do these abilities change? Before we can begin to ask *why* spatial abilities and math performance are related, we need to know a lot more about *how* they are related.

TWO TYPES OF IMAGINARY PERSPECTIVE-TAKING

Based on the previous research, Everett, Croft and Flavell (1981) proposed and confirmed the distinction of two specific perspective-taking abilities, distinguishing two levels of children's competencies. Level 1 competence concerns the visibility of objects, i.e. the ability to conclude which objects are visible and which are not from a certain point of view. Level 2 competence is related to the appearance of objects, that is, the ability to make a judgment about how the object looks from a certain point of view. Van den Heuvel-Panhuizen, Elia and Robitzsch (2015) in their research on imaginary perspective-taking (IPT) of preschoolers take the terms and talk about IPT type 1 (visibility) and IPT type 2 (appearance).

The aim of the study by Van den Heuvel-Panhuizen, Elia and Robitzsch (2015) was to gain a better insight into the IPT of preschoolers, especially the developed abilities of IPT type 1 (visibility) and IPT type 2 (appearance) and cross-cultural patterns in this competence including children from two countries, the Netherlands and Cyprus. Specifically, Van den Heuvel-Panhuizen, Elia and Robitzsch investigated the extent to which preschool children developed IPT type 1 and type 2 abilities, how these competencies are related, and whether IPT competency is related to children's age, mathematical abilities, and gender. The sample consisted of four and five-year-old children in the Netherlands (N = 334) and Cyprus (N = 304). Children's IPT competence was assessed with a paper-and-pencil test of different pictorial representations of objects that take perspective into account and require IPT type 1 or IPT type 2. Figure 2 shows two items from the test. The Duck item (instruction: "The duck has fallen into the hole. He looks up. What does he see?") is intended for measuring IPT type 1, while the Soccer item (instruction: "Two children are playing soccer. How do you see it if you look from above like a bird?") intended for measuring IPT type 2.

Figure 2. Two items: a) Duck item and b) Soccer item to measure IPT type 1 and type 2 respectively (Appendix)Figure 2



The results showed interesting common patterns for the two IPT types in both countries. Specifically, IPT type 2 items were significantly more difficult to solve than IPT type 1 items, and children's achievements on the first items imply achievements on the latter. Also, in both countries, IPT type 1 appeared to develop during preschool years. For IPT type 2 this was the case only in the Netherlands. There were no significant gender differences in IPT competencies among the preschoolers in the two countries. However, the relationship between children's IPT competence and mathematical abilities was not so clear.

Attention to spatial abilities is mentioned in the relevant papers of Mammana and Villani (1998), NCTM (2000) and NRC (2009), as well as abilities in specifying locations of objects (including interpreting relative positions in space) and using visualization (including creating mental images of geometric objects using spatial memory and spatial visualization, as well as recognition and representation of objects from different perspectives). Similar approaches can be found in curricula in England (Department of Education 2013), Australia (Board of Studies New South Wales 2012), the Netherlands (Van den Heuvel-Panhuizen, Buys 2008), and Cyprus (Cyprus Ministry of Education and Culture 2010). In Serbia, such approaches in mathematics curricula are not sufficiently recognized and, therefore, represent potentially unequal opportunities for learning and developing spatial ability and spatial reasoning ability (Pravilnik o planu nastave i učenja za prvi ciklus osnovnog obrazovanja i vaspitanja i programu nastave i učenja za prvi razred osnovnog obrazovanja i vaspitanja 2017; Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja 2018; Pravilnik o programu nastave i učenja za treći razred osnovnog obrazovanja i vaspitanja 2019; Pravilnik o programu nastave i učenja za četvrti razred osnovnog obrazovanja i vaspitanja 2019). Here are the results of the analysis of the mentioned documents (mathematics curriculum) for the Space area in the Geometry domain:

1. In the first grade for seven-year-old children, outcome states that they are able to determine the mutual position of objects and beings and their positions in relation to the ground, as well as to notice and name the geometric shapes of objects in the immediate environment.
2. In the second and third grades, the outcomes that explicitly refer to the development of spatial thinking are not stated.
3. In the fourth grade, the outcome related to the recognition of the pictorial representation of the body viewed from different sides is stated.

At preschool age, children are focused on exploring the space around them, that is, in a real environment (Opšte osnove predškolskog programa 2006; Osnove programa predškolskog vaspitanja i obrazovanja 2018). The question arises as to how many educators put them in different situations that require positions of perception and description of different spatial situations.

From pre-school onwards, children should be provided with learning opportunities to further develop their spatial skills. This is an indicator that children in the continuity of mathematics education through curricula in Serbia are not clearly supported in the development of spatial reasoning, including the spatial ability of imaginary perspective-taking.

METHODOLOGY

The subject of this research is two types of imaginary perspective-taking as a special spatial component that develops from an early age. The research was conducted in May 2022. A total of 88 children, divided into two groups, participated in the study: preschool children (31) and primary second-grade pupils (57). All participants are from urban areas. This research aimed to examine the level of imaginary perspective-taking ability in children aged six to eight years old. In reference to this goal, research questions arise:

1. What is the general performance of the preschool children and second grade pupils on the imaginary perspective-taking test?
2. a) Are children in both age groups, preschool and primary school, equally successful on an imaginary perspective-taking test and parts of this test? b) Are the participants of a certain age group equally successful at two types of imaginary perspective-taking IPT1: *visibility*, or which objects are visible, and IPT2: *appearance,* or how an object or situation looks from a certain point of view.
3. What is the general performance of the children on individual tasks on the imaginary perspective-taking test?
4. Are the preschoolers from Serbia equally successful as their peers from the Netherlands and Cyprus on the imaginary perspective-taking test and the parts of the test?

To address the research questions a descriptive method and paper and pencil test were used. The imaginary perspective-taking test consists of 13 tasks, out of which seven tasks refer to IPT1, visibility, and six refer to IPT2, appearance. Each task had four possible answers and participants could reach a maximum of 13 points. The instrument was taken from the research of Van den Heuvel-Panhuizen, Elia and Robitzsch (2015) with little technical adaptations and as such it is presented in the Appendix. We calculated the reliability of IPT items by using Cronbach's alpha. The reliability of 13 items was $\alpha = 0.64$. The identified reliability is considered good, but below the frequently used minimal criterion of 0.70. However, given the heterogeneous nature of IPT, such low reliability can be expected (Cortina, 1993).

For data processing, we used SPSS 22. The general performances of the participants were analyzed using descriptive statistical measures. In order to examine if the test scores have normal distribution, we used the Shapiro–Wilk normality test. Intending to determine the statistical differences between the preschool age group and primary school age group in an imaginary perspective-taking test, we ran the Mann–Whitney U test. For determining the statistical differences in individual items between the groups we conducted Pearson's Chi-square test of homogeneity. Further, to examine if the participants are equally successful in both IPT types we used the paired sample t-test for the preschool group and Wilcoxon Signed Rank Test for the primary school group of pupils. By using One-sample t-test we wanted to compare Serbian preschool children's success with the success of the Netherlands and Cyprus children (Van den Heuvel-Panhuizen, Elia, Robitzsch 2015).

RESULTS AND DISCUSSION

The first research question refers to participants' general performance on the imaginary perspective-taking test. Children could reach the maximum of 13 points on the test. The results are shown in Table 1.

Table 1. General performance of the participants on the IPT test.

age group	*N*	*M*	*SE*	*SD*	*Mdn*	Range[a)]	Mod[a)]	Shapiro-Wilk
preschool	31	0.59	0.03	0.16	0.62	8	7 and 9	0.97*
second-grade	57	0.83	0.02	0.14	0.84	6	11	0.89**

*p = 0.41
**p = 0.00
[a)] calculated for number of points on the test

The results show the mean score or proportion of correct answers. The mean score of correct answers for preschool children is $M = 0.59$ ($SD = 0.16$), and for second-grade pupils it is $M = 0.83$ ($SD = 0.14$). Although the mean score of the second-grade participants indicates that the test was easy for them, the range of six points shows us that not all of the students developed IPT ability. Shapiro-Wilk normality tests show that preschool children's performance follows normal distribution, but that is not the situation in the group of the second-graders. This might indicate that the test is not suitable for students older than preschool age, but with further analysis we come to some conclusions and insights.

Further, we aimed to a) compare the preschool children's success with the primary school pupils' success on the IPT test and parts of the test; b) explore if participants of a certain age group are equally successful in both IPT types (IPT1, visibility and, IPT2, appearance).

From general performance results we have seen that older participants were expectedly better than the younger ones, so we wanted to explore with the Mann–Whitney U test if those differences are statistically significant. The rank test confirmed our doubt that the difference between the preschool children ($Mdn = 0.62$, $n = 31$) and the second-grade pupils ($Mdn = 0.85$; $n = 57$), $U = 240$, $z = -5.67$, $p = 0.00$, $r = 0.6$) is statistically significant and the calculated effect size is considered large. A significant outperformance was found on the parts of the test that is on IPT1 visibility items $U = 285.50$; $z = -5.39$, $p = 0.00$; $r = 0.6$ ($Mdn = 0.57$, $n = 31$ for preschoolers and $Mdn = 0.86$; $n = 57$ for second graders) and IPT2 appearance items $U = 409$; $z = -4.24$, $p = 0.00$; $r = 0.5$ ($Mdn = 0.50$; $n = 31$ for preschoolers and $Mdn = 0.83$; $n = 57$ for second graders).

Considering that older participants outperformed the younger ones, we were curious to explore how IPT ability develops within age groups according to two types of this ability. The paired sample t-test has shown that preschool children were equally successful in the IPT1 ($M = 0.60$, $SD = 0.23$) and in the IPT2 items ($M = 0.58$, $SD = 0.18$), $t(30) = 0.395$, $p = 0.696$. With school age participants' situation is slightly different. Wilcoxon Signed Rank Test indicates that second grade students are significantly better in IPT1 items ($Mdn = 0.86$) then in IPT2 items ($Mdn = 0.83$) $z = -2.670$, $p = 0.008$, $r = 0.25$. However, this difference is considered small.

Within two types of imaginary perspective-taking, we examined the participants' success in individual items. The first seven items are IPT1 visibility items and the rest of the items are IPT2 appearance items. The results are shown in Table 2.

Table 2. Participants' success on individual IPT items.

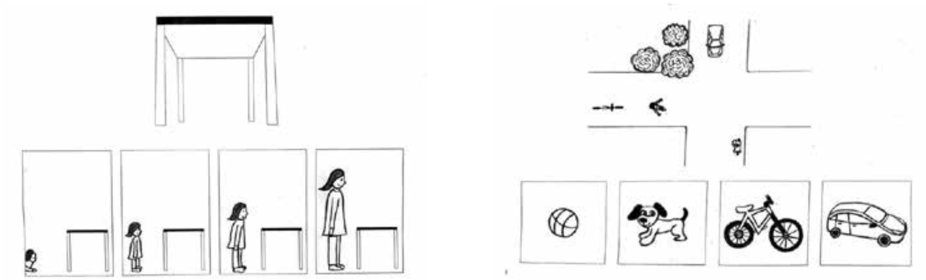
Task	Items	preschool age	school age second grade	χ^2 (1, N = 88)[a)]
		%	%	
1	IPT1 Umbrella	77.4	73.3	0.017
2	IPT1 Duck	77.4	87.7	0.915
3	IPT1 Crossing	25.8	78.9	21.505*
4	IPT1 Basket	54.8	98.2	23.774*
5	IPT1 Tower	48.4	93.0	20.270*
6	IPT1 Wall	80.6	98.2	6.262*
7	IPT1 Hole	54.8	78.9	4.508*
8	IPT2 Mouse	71.0	89.5	3.643
9	IPT2 Cucumber	77.4	73.7	0.017
10	IPT2 Fence	45.2	78.9	8.901*
11	IPT2 Soccer	80.6	77.2	0.011
12	IPT2 Table	29.0	77.2	17.484*
13	IPT2 Tree	45.2	73.7	5.881*

[a)] Yates correction was used for 2x2 tables.
* $p < 0.05$

The result of the Pearson's Chi-square of homogeneity indicates differences in success between the two age groups in the following tasks: Crossing, Basket, Tower, Wall, Hole (IPT1), Fence, Table, and Tree (IPT2). As the second-graders outperformed the younger participants in five of the mentioned items that are

IPT1 items, we can see, once again, that the second-graders are not as successful in IPT2 items as in IPT1 items. The preschoolers achieved better results in Umbrella, Cucumber, and Soccer tasks, but this is insignificant. We noticed that preschool participants are more successful in visibility items that are posing bird's or bottom view perspectives such as in Umbrella and Duck (77.4% success in both items), but less successful in visibility tasks that involve someone else's perspective such as in Crossing, Basket, Hole or Tower (25.8%, 54.8%, 54.8%, 48.4% respectively). In the latter items, we have an angled view, which might be slightly difficult for younger children. Similarly, in appearance items that include bird's perspective, such as item Mouse,(71%), the younger participants were remarkably successful. In this task, a front view of the mouse is drawn, and children have to choose between one of four drawings that shows the mouse from a bird's eye perspective (Mouse and other tasks are available in Appendix). The preschoolers have better performance in Mouse task than in Table (29%) which is also an appearance item (see Figure 3). As Figure 3 shows, children should imagine how each of the four girls sees the table and choose the correct answer. We have not noticed such regularity in the second-graders' performances. The older students outperformed the younger ones in the tasks that require some experience. For example, look at the Crossing item (Figure 3). Second-graders are, we assume, more independent in traffic than preschoolers. In this task the children have to choose what the boy sees while walking down the street. The second-grade pupils demonstrated a high performance in every item, but it seems that IPT1 Umbrella (73.3%), IPT2 Cucumber (73.7%) and Tree (73.7%) are items that are more challenging for them. In the Umbrella task, the girl is holding an umbrella, a flower, and a ball under it. Children should choose what is seen from a bird's eye perspective (the girl's head, the flower, the ball or the surface of the umbrella). In the cucumber task, children should choose the appropriate picture that represents the cross section of a cucumber according to the picture that shows a knife cutting the cucumber. In Tree task, a tree is shown upside down with a mouse hanging on a branch. Children should choose what the tree would look like in reality. We consider that the Tree item is more complex than other tasks because of the given answers that include mental rotation. An explanation for Cucumber task can

Figure 3. Two items: a) Table to measure IPT 2 and b) Crossing to measure IPT 1 (Appendix).



be that when we cut a cucumber, a cross-section shape is a circle, but perhaps it is not so in pupils' experience nowadays (chopping machines are used).

We have analyzed which incorrect answers the participants usually chose, and we have some interesting insights. First, we point out that in most of the tasks the preschoolers have chosen more incorrect answers than the second-graders. The most frequently chosen incorrect answer by the children of this age was the answer c) in item Table (see Appendix, IPT 2 – Table), where as many as 41.9% of children considered this to be the correct answer. Answer c) in item Table might be the most natural position for children of five to six years, but they did not consider the instruction that points out a concrete picture of the table. Among the second-grade pupils, the common incorrect answers appeared in two tasks: Cucumber (answer c), 21.1%) (see Appendix, IPT 2 – Cucumber), and Tree (answer d), 21.1%), (see Appendix, IPT 2 – Tree). The incorrect answer c) in the Cucumber item shows an intersection as the correct answer (instead of answer b), so the pupils who were wrong might have been rushing or did not understand the instruction. The incorrect answer b) in the Tree item is similar to the correct answer d), but the animal on the tree is on the wrong side of the tree. A greater number of errors by the second-graders is also visible in the Umbrella task, where more pupils gave an incorrect answer b) (15.8%) and c) (10.5%) (see Appendix, IPT 1 – Umbrella, correct d) and in the Soccer task (answer b), 10.5%) (see Appendix, IPT 2 – Soccer, correct c). Based on the quantitative data, we can see that the children had the lowest number of incorrect answers in the Wall task in which a wall is placed between two children, and the question is in which situation the children can see each other depending on the height of the wall between them (see Appendix). This may imply that the situation presented in this task is intuitively and experientially closest to them. However, based on the data, we cannot come to a single conclusion as to why children choose a certain situation, that is, a picture as one of the incorrect answers. For that kind of conclusion, an interview would be a more suitable technique.

Finally, we wanted to compare the success of the preschool children in Serbia to the success of the children from Cyprus and the Netherlands from existing research (Van den Heuvel-Panhuizen, Elia, Robitzsch 2015). The one-sample t-test showed that the preschool children from our sample showed similar scores on the imaginary perspective-taking test as the children from the Netherlands (age = 5.69), $M = 0.59$, $SD = 0.16$, $t(30) = -0.327$, $p = 0.746$ (the tested value from the cited research is 0.60, that is, an average of 60% success on the test). However, compared to the children from Cyprus (age = 5.61), the preschool children from Serbia have a significantly higher score on the imaginary perspective-taking test $M = 0.59$, $SD = 0.16$, $t(30) = 3.833$, $p = 0.001$ (the tested value from the cited research is 0.48, i.e. 48% success rate on the test). We further compared the performance on the parts of the test, namely, on the tasks of imaginary perspective-taking type 1 and imaginary perspective-taking type 2 between the countries. The children from the Netherlands are significantly better on the IPT1 ($M = 0.74$) compared to the preschool children from our sample (*M* = 0.60, *SD* = 0.23), *t*(30) = -3.432, *p* = 0.002, but the children from Serbia are significantly more successful on IPT2 tasks (*M* = 0.58, *SD* = 0.18), *t*(30) = 3.800, *p* = 0.001 compared to the children from the Netherlands (*M* = 0.46). There is no difference in performance on IPT1 items between the children from Cyprus (*M* = 0.61) and children from our sample (*M* = 0.60, *SD* = 0.23), *t*(30) = -0.266, *p* = 0.792, but there is on IPT2 items in favor of the children from Serbia (*M* = 0.58, *SD* = 0.18), *t*(30) = 7.580, *p* = 0.00 (*M* = 0.34 mean score of children from Cyprus).

CONCLUSION

The results of the research on children's success in perspective-taking tasks show that second-grade students are more successful than preschool children, and we can conclude that as children grow older, their ability to take a perspective in a two-dimensional environment improves. The children of the second grade were more successful in relation to the preschool children and in relation to both types of perspective-taking abilities (IPT1 and IPT2), with deviations on specific perspective-taking tasks of type 1 (Umbrella) and tasks of type 2 (Cucumber and Soccer) where the preschoolers were more successful. This can be explained by the fact that the younger participants, often carried in the arms of their parents, had the opportunity to perceive objects from a bird's eye view and were extremely good at tasks of this type. On the other hand, upon enrolling at primary school, this ability was not further developed and nurtured, as evidenced in the mathematics curriculum (Pravilnik o planu nastave i učenja za prvi ciklus osnovnog obrazovanja i vaspitanja i programu nastave i učenja za prvi razred osnovnog obrazovanja i vaspitanja 2017; Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja 2018), so the second-grade students have somewhat lower scores. Observing only a specific type of perspective-taking ability, we came to a conclusion that preschool children are equally successful in both types of perspective-taking, while second-grade pupils are somewhat more successful on visibility tasks (IPT1) compared to appearance tasks (IPT2). The analysis of the most frequently chosen incorrect answers showed us that children of preschool age and second grade have similar misconceptions, that is, difficulties when they are put in situations to determine whether an object is visible or how it looks from another perspective. However, these quantitative indicators cannot shed light on why children think that a certain answer, i.e. a picture, is more appropriate to the given situation. Some future research may address this issue. Since in the research we relied on the existing research instrument (Van den Heuvel-Panhuizen, Elia, Robitzsch 2015), we were also interested in the position of the preschool children from Serbia in relation to the children of the same age in Cyprus and the Netherlands. The results of our research showed that Serbian children from our sample are significantly better at solving perspective-taking tasks than the children from Cyprus sample, and equally successful as the children from the Netherlands sample. More precisely, when it comes to the types of perspective-taking abilities, the children from our sample are significantly more successful in the tasks referring to how something looks (IPT2), compared to the children from the samples from both countries, but less successful in the tasks of whether something can be seen (IPT1) compared to Dutch children and equally successful as Cyprus children. However, we emphasize that the sample in our research is small, and that further analyses of imaginary perspective-taking abilities should be conducted on a larger sample (both rural and urban areas), and the conclusions of such analyses can be generalized for a wider population.

Finally, we will refer to the instrument we used in the research itself, taken from the mentioned research (Van den Heuvel-Panhuizen, Elia, Robitzsch 2015). As noted, the test itself challenged preschoolers, while it was easy for the second-graders. In addition, we noticed certain shortcomings in certain tasks. For example, the Fence task did not send a clear message about which answer was correct since one of the first two answers offered could be considered the correct answer. Also, with the Tower task, it is not entirely clear in which direction the girl is looking, as well as whether certain objects can create a "distraction" or obscure how visible they are from a certain position. Furthermore, the Tree task also did not clearly depict the appearance of the object under consideration in the picture. Certain technical changes were made in the mentioned tasks. During the realization of the research, and based on the questions asked by the pupils of the second grade, we also observed that the task involving streets was unclear to them, namely, that based on the picture, they could not see in which direction the child in the picture was moving.

The aforementioned research results indicate that children of preschool and primary school age need to be placed in situations where they will have an opportunity to consider the visibility or appearance of a certain object from another perspective, especially as there are few such requirements in the preschool and primary school curricula, and there is a growing need to point out the importance of geometry and space (Sinclair, Bruce 2014; Uttal, Cohen 2012; Đokić 2018; Đokić, Boričić, Jelić 2022). In addition, we emphasize the need to examine in the future children's perspective-taking ability at other ages with an adequate instrument (such as in Frick Möhring, Newcombe 2014), as well as in other settings.

REFERENCES

Björklund, van den Heuvel-Panhuizen, Kullberg (2020): C. Björklund, M. van den Heuvel-Panhuizen, A. Kullberg, Research on early childhood mathematics teaching and learning, *ZDM Mathematics Education*, 52/4, Germany: Springer Verlag, 607–619.

Board of Studies New South Wales (2012): *NSW syllabus for the Australian curriculum Mathematics K–10 syllabus*. Sydney: Author.

Bruce, Davis, Sinclair, McGarvey, Hallowell, Drefs, Francis, Hawes, Moss, Mulligan, Okamoto, Whiteley, Woolcott (2017): C. D. Bruce, B. Davis, N. Sinclair, L. McGarvey, D. Hallowell, M. Drefs, K. Francis, Z. Hawes, J. Moss, J. Mulligan, Y. Okamoto, W. Whiteley, G. Woolcott, Understanding gaps in research networks: using "spatial reasoning" as a window into the importance of networked educational research, *Educational Studies in Mathematics*, 95/2, Netherlands: Springer Netherlands, 143–161.

Bryant (2008): P. Bryant, Paper 5: Understanding spaces and its representation in mathematics, In: T. Nunez, P. Bryant, A. Watson (Eds.), *Key understanding in mathematics learning: A report to the Nuffield Foundation*, London: Nuffield Foundation.

Clements, Battista (1992): D. H. Clements, M. T. Battista, Geometry and spatial reasoning, In: D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, Stuttgart: Macmillan, 420–464.

Cortina (1993): J. M. Cortina, What is coefficient alpha? An examination of theory and applications, *Journal of Applied Psychology*, 78/1, USA: American Psychological Association, 98–104.

Cyprus Ministry of Education and Culture (2010): *Program of studies in mathematics*, Nicosia: Cyprus Ministry of Education and Culture.

Davis, Spatial Reasoning Study Group (Eds.) (2015): B. Davis, Spatial Reasoning Study Group (Eds.), *Spatial reasoning in the early years. Principles, assertions, and speculations*, United Kingdom: Routledge.

Department for Education (2013): *Mathematics programmes of study: Key stages 1 and 2. National curriculum in England*, England: Department for Education.

Đokić (2018): O. J. Đokić, Research perspectives of initial geometry education, *Journal of Pedagogical Research*, 2/2, Turkey, 102–111.

Đokić, Boričić, Jelić (2022): O. J. Đokić, M. M. D. Boričić, M. S. Jelić, Comparing ICT with Physical Manipulative Supported Learning of 3D Geometry in Elementary School, *Journal of Educational Computing Research*, 59/8, USA: SAGE Publications Inc., 1623–1654.

Everett, Croft, Flavell (1981): B. A. Everett, K. Croft, E. R. Flavell, Young children's knowledge about visual perception: Further evidence for the Level 1 – Level 2 distinction, *Developmental Psychology*, 17/1, USA: American Psychological Association, 99–103.

Freudenthal (1973): H. Freudenthal, *Mathematics as an educational task*, Dordrecht, The Netherlands: D. Reidel Publishing Company.

Frick, Möhring, Newcombe (2014): A. Frick, W. Möhring, N. S. Newcombe, Picturing perspectives: development of perspective-taking abilities in 4- to 8-year-olds, *Frontiers in Psychology*, 5/386, Lausanne, Switzerland: Frontiers Media.

Guay, McDaniel (1977): R. B. Guay, E. D. McDaniel, The relationship between mathematics achievement and spatial abilities among elementary school children, *Journal for Research in Mathematics Education*, 8/3, USA: National Council of Teachers of Mathematics, 210–215.

Ishikawa, Newcombe (2021): T. Ishikawa, N. S. Newcombe, Why spatial is special in education, learning, and everyday activities, *Cognitive Research: Principles and Implications*, 6/20, USA: Springer Open.

Lubinski (2010): D. Lubinski, Spatial ability and STEM: A sleeping giant for talent identification and development, *Personality and Individual Differences*, 49/4, Amsterdam, Netherlands: Elsevier, 344–351.

Mammana, Villani (Eds.) (1998): C. Mammana, V. Villani, *Perspectives on the teaching of geometry for the 21st century – An ICMI Study*, Amsterdam: Kluwer Academic Publishers.

Mix, Cheng, (2012): K. S. Mix, Y. L. Cheng, The Relation Between Space and Math: Developmental and Educational Implications, In: J. B. Benson (Ed.), *Advances in child development and behavior*, 42, Amsterdam: Elsevier Inc.,197–243.

National Council of Teachers of Mathematics (NCTM) (2000): *Principles and standards for school mathematics*, Reston, VA: National Council of Teachers of Mathematics.

National Research Council (NRC) (2009): *Mathematics learning in early childhood: Paths toward excellence and equity*, USA: The National Academies Press.

Newcombe (1989): N. Newcombe, The development of spatial perspective taking, In: H. W. Reese (Ed.), *Advances in child development and behavior*, 22, New York: Academic Press, 203–247.

Newcombe, Huttenlocher (1992): N. Newcombe, J. Huttenlocher, Children's early ability to solve perspective-taking problems, *Developmental Psychology*, 28/4, USA: American Psychological Association, 635–643.

Newcombe, Huttenlocher (2006): N. S. Newcombe, J. Huttenlocher, Development of Spatial Cognition, In: D. Kuhn, R. Siegler (Eds.), *Handbook of Child Psychology* (6th ed.), 2, New Jersey: John Wiley & Sons, Inc., 734–776.

Newcombe (2010): N. Newcombe, Picture this: Increasing math and science learning by improving spatial thinking, *American Educator*, 34/2, USA: American Federation of Teachers, 29–35.

Newcombe, Uttal, Sauter (2013), N. S. Newcombe, D. H. Uttal, M. Sauter, Spatial development, In: P. Zelazo (Ed.), *Oxford handbook of developmental psychology*, 1, New York: Oxford University Press, 564–590.

Ontario Ministry of Education (2005): *Grade 6 mathematics curriculum*, Toronto: Ontario Education Ministry.

Pravilnik o opštim osnovama predškolskog programa (2006): *Prosvetni glasnik*, br. 14.

Pravilnik o osnovama programa predškolskog vaspitanja i obrazovanja (2018): *Prosvetni glasnik*, br.16.

Pravilnik o planu nastave i učenja za prvi ciklus osnovnog obrazovanja i vaspitanja i programu nastave i učenja za prvi razred osnovnog obrazovanja i vaspitanja (2017): *Prosvetni glasnik*, br. 10.

Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja (2018): *Prosvetni glasnik*, br. 16.

Pravilnik o programu nastave i učenja za treći razred osnovnog obrazovanja i vaspitanja (2019): *Prosvetni glasnik*, br. 5.

Pravilnik o programu nastave i učenja za četvrti razred osnovnog obrazovanja i vaspitanja (2019): *Prosvetni glasnik*, br. 11.

Sinclair, Bruce (2014): N. Sinclair, C. D. Bruce, Spatial reasoning for young learners, In: P. Liljedahl, C. Nicol, S.Oesterle, D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA*, 36/1, Vancouver, Canada: PME, 173–203.

Uttal, Cohen (2012): D. H. Uttal, C. A. Cohen, Spatial thinking and STEM education: When, why and how?, In: B. Ross (Ed.), *Psychology of learning and motivation*, 57, New York: Academic Press, 148–178.

Uttal, Meadow, Tipton, Hand, Alden, Warren, Newcombe (2013): D. H. Uttal, N. G. Meadow, E. Tipton, L. L. Hand, A. R. Alden, C. Warren, N. S. Newcombe, The malleability of spatial skills: A meta-analysis of training studies, *Psychological Bulletin*, 139/2, USA: American Psychological Association, 352–402.

Van den Heuvel-Panhuizen, Buys (Eds.) (2008): M. Van den Heuvel-Panhuizen, K. Buys, *Young children learn measurement and geometry*, Rotterdam: Sense.

Van den Heuvel-Panhuizen, Elia, Robitzsch (2014): M. Van den Heuvel-Panhuizen, I. Elia, A. Robitzsch, Paper 6: Kindergarteners' abilities in perspective taking, In: P. Liljedahl, C. Nicol, S. Oesterle, D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA*, 36/1, Vancouver, Canada: PME, 184-185.

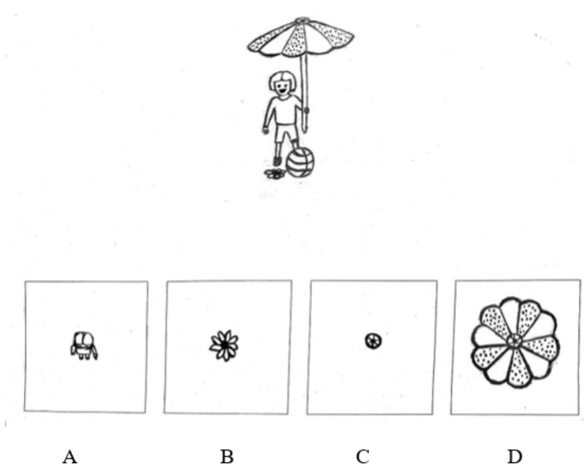
Van den Heuvel-Panhuizen, Elia, Robitzsch (2015): M. Van den Heuvel-Panhuizen, I. Elia, A. Robitzsch, Kindergartners' performance in two types of imaginary perspective-taking, *ZDM Mathematics Education*, 47/3, Germany: Springer Verlag, 345–362.

Wai, Lubinski, Benbow (2009): J. Wai, D. Lubinski, C. P. Benbow, Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance, *Journal of Educational Psychology*, 101/4, USA: American Psychological Association, 817–835.

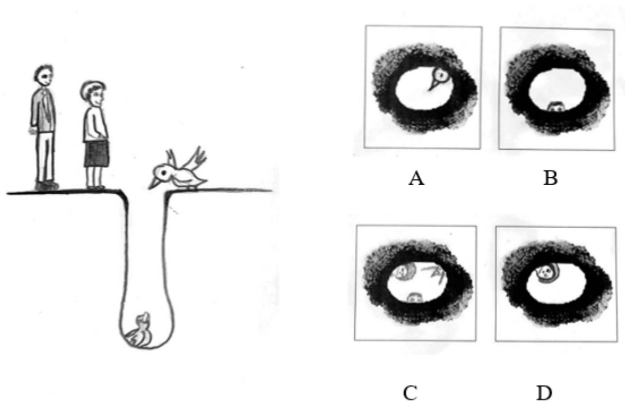
Young, Levine, Mix (2018): C. J. Young, S. C. Levine, K. S. Mix, The Connection between Spatial and Mathematical Ability across Development, *Frontiers in Psychology*, 9/755, Lausanne, Switzerland: Frontiers.

APPENDIX

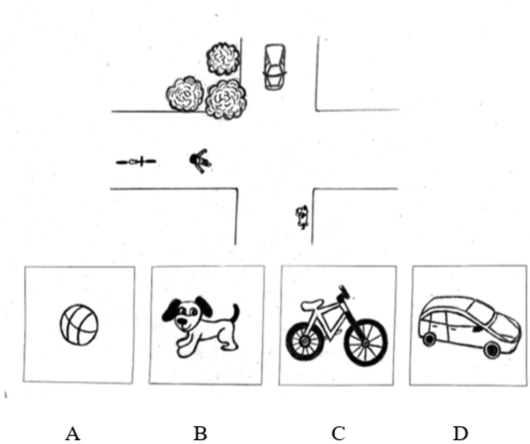
IPT1 – UMBRELLA



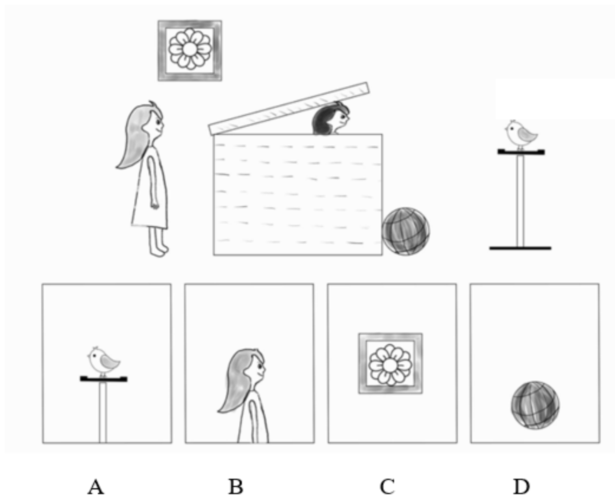
IPT1 – DUCK



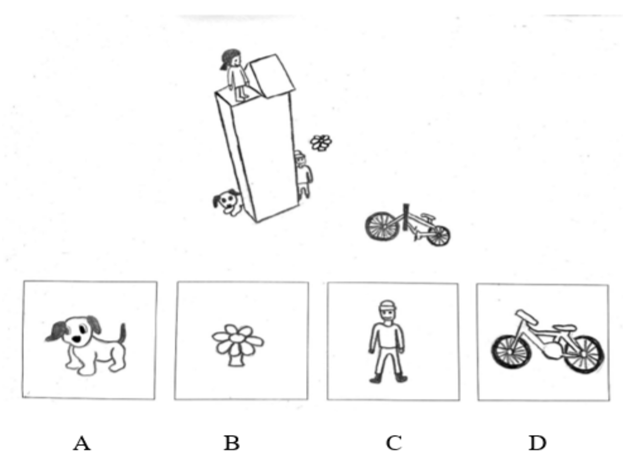
IPT – CROSSING



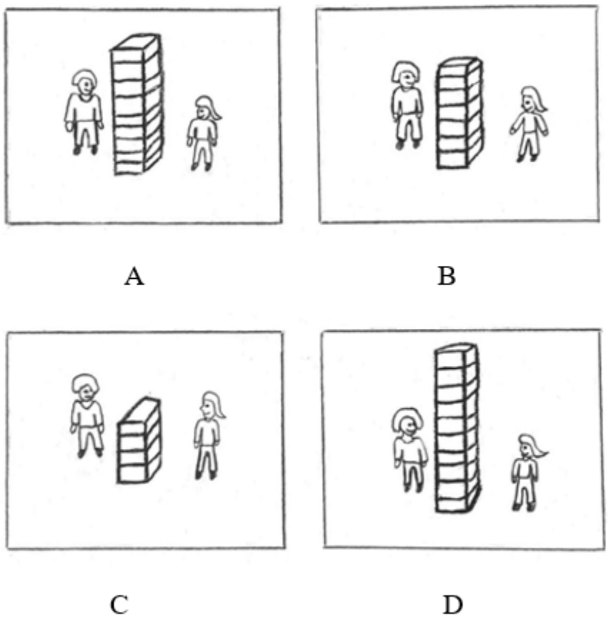
IPT 1 – BASKET



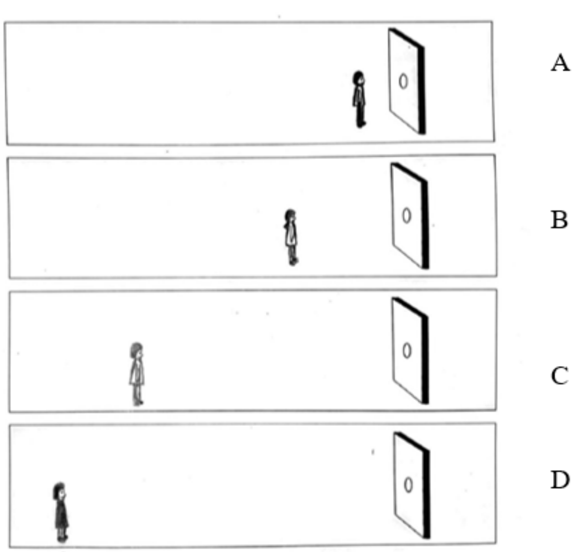
IPT 1 – TOWER



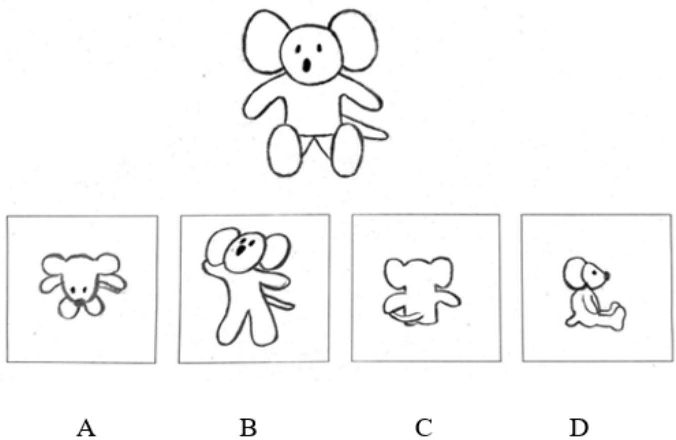
IPT 1 – WALL



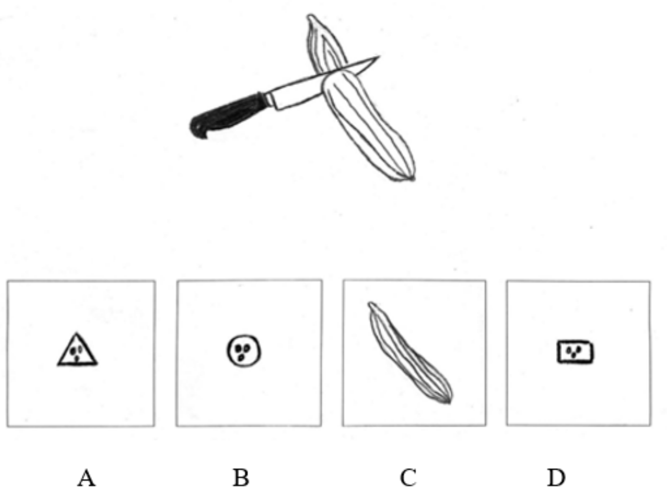
IPT 1 – HOLE



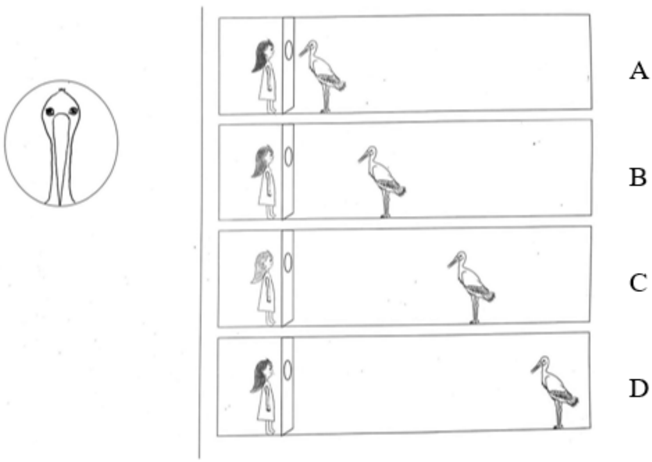
IPT 1 – MOUSE



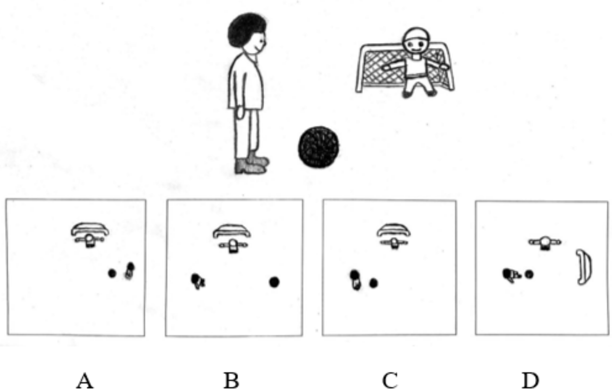
IPT 2 – CUCUMBER



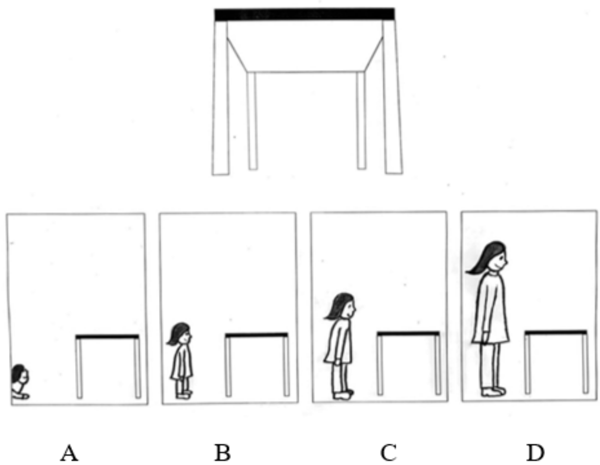
IPT 2 – FENCE



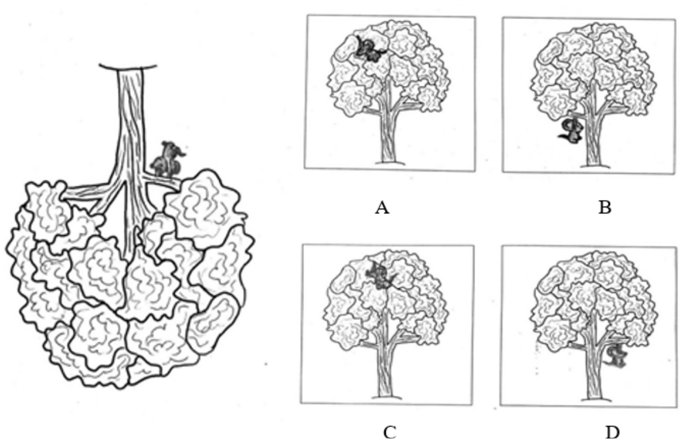
IPT 2 – SOCCER



IPT 2 – TABLE



IPT 2 – TREE



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СПОСОБНОСТ ЗАУЗИМАЊА ПЕРСПЕКТИВЕ КОД ДЕЦЕ ПРЕДШКОЛСКОГ И МЛАЂЕГ ШКОЛСКОГ УЗРАСТА

Резиме: Иако просторна способност представља битан елемент процеса учења и поучавања у математици, није свака компонента ове способности добила довољно истраживачке пажње. У овом раду се фокусирамо на способност деце предшколског узраста и ученика другог разреда основне школе при решавању две врсте проблема са заузимањем перспективе датих у облику теста (тест папир–оловка). Резултати показују разлику у решавању проблема сагледавањем заузимања перспективе између деце предшколског узраста и ученика другог разреда који чине узорак у овом истраживању. Осим тога, резултати показују да ученици другог разреда нису у потпуности развили ову посебну просторну способност, будући да су нешто мање успешни у задацима изгледа него задацима видљивости (видљивост и изглед представљају посебне способности заузимања перспективе). Приметили смо индивидуалне разлике у обе старосне групе. Поред тога, узорак деце предшколског узраста из Србије подједнако је успешан као деца из Холандије и значајно је бољи од деце са Кипра на узорку истог узраста који наводе Ван ден Хувел-Панхуизен, Елиа и Робич (Van den Heuvel-Panhuizen, Elia and Robitzsch 2015). Општи закључак и образовна импликација је да способност заузимања перспективе треба даље неговати у раном образовању и код деце млађег школског узраста.

Кључне речи: заузимање перспективе, видљивост, изглед, просторно резоновање, предшколска деца и деца млађег школског узраста, Србија.