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*The 28th International Scientific Conference
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**THE STATE
PROBLEMS
AND NEEDS
OF THE MODERN
EDUCATION
COMMUNITY**

BOOK OF PROCEEDINGS

Editors

Jelena STEVANOVIĆ

Dragana GUNDOGAN

Branislav RANĐELOVIĆ



Российский университет
дружбы народов



Belgrade, 2022.

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“Educational Research and School Practice”

The State, Problems, and Needs of the Modern Education Community

December 9th, 2022
Belgrade

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**MODERN
APPROACHES
TO LEARNING
AND TEACHING**

YOUNG PUPILS' INTUITIVE UNDERSTANDING AND STRATEGIES OF AREA MEASUREMENT²⁰

Olivera J. Đokić²¹

University of Belgrade, Teacher Education Faculty, Republic of Serbia

Neda D. Osmokrović

Universität Wien, Bildungswissenschaft, Austria

Introduction

Learning area measurement in mathematics instruction implies certain phases that entail rhetorical and symbolical generalizations in terms of mathematical formulas (Smith III & Barrett, 2017; Zacharos, 2006; Zeljić & Ivančević, 2019). It is through mathematical formulas that geometry is represented in Serbian mathematics curricula and this fact is the starting point of our research.

When pupils have to solve mathematical tasks, they use various strategies that differ in terms of the correctness of their solution, the time needed for completing the task, and task requirements and scope (Siegler, 1991).

To measure the area of a rectangular and overcome the problems arising from pupils' misunderstanding of the area formula, it is recommended to take a closer look at the structure of the rectangular array by covering the area of the rectangular with a mathematical manipulative in the form of a unit of measurement that pupils are intuitively familiar with from the onset (Đokić, 2014, 2017; Van de Walle, Karp &

²⁰ *Note.* This research is a result of a modified and, in the theoretical segment, amended master's thesis *Intuitivno razumevanje mera i merenja površine i strategije koje učenici koriste pri izračunavanju površine [Intuitive understanding of measures and area measurements and strategies students use to calculate area]* defended at the Teacher Education Faculty of the University of Belgrade on July 1, 2019 (mentor Olivera J. Đokić, PhD, Associate Professor).

²¹ E-mail: olivera.djokic@uf.bg.ac.rs

Bay-Williams, 2013). The concept of covering would enable pupils to conceptualize the relationship between the unit's dimensions and the dimensions of the rectangular. After this phase, through length measurement and multiplication, pupils can solve the area measurement task using mathematical formulas with understanding.

Apart from the covering strategy, the paper looks at the ways pupils conceptualize area (Outhred & Mitchelmore, 2000; Reynolds & Wheatley, 1996; Nunes, Light & Mason, 1993). Can an actual misunderstanding of the structure of a rectangular array be found in our mathematics curricula and, if so, can it be overcome by applying the idea of covering the area of a rectangular array?

We conducted an empirical study on pupils' strategic approaches to covering the area of a rectangular in order to understand how pupils calculate its area. We identified the strategies used by pupils and examined their stages of development.

Method

The research was conducted in 2019 on a random sample of 44 pupils from two primary schools in Belgrade located in a medium socio-economic area. The sample comprised 24 girls and 20 boys from five classes, aged 6-9, including 13 pupils of preschool age, 12 first-grade pupils, 8 second-grade pupils, and 11 third-grade pupils (Osmokrović, 2019).

The data were collected by conducting a 15-minute interviews and observing the ways pupils measured the area of a rectangular with the aim of determining which strategies were used. The sample was selected based on the fact that the concept of area is not taught in this age group.

This empirical study *aimed* to identify pupils' strategies for calculating the area of a rectangle and classify the identified strategies into corresponding stages of development.

The research objectives were as follows:

1. To identify the pupils' strategies for measuring the area of a rectangular array.
2. To classify the identified strategies into developmental levels.
3. To identify the ways in which pupils conceptualize area.

The measuring instrument comprised three tasks in a linked interview, taken from the research conducted by Outhred and Mitchelmore (2000) (see Appendix 1; the interviews was conducted by the second author of this paper). The pupils used a piece of paper, a pencil, a ruler, and a manipulative in the form of a cardboard square. *The first task* examined pupils' strategies by using one physical unit in the form of a 2 cm cardboard square, which enabled pupils to solve the task without formal measuring. Participants were asked to determine how many such square tiles could fit into a bigger, 8 cm square drawn on a piece of paper. In *the second task*, pupils were shown the picture of the rectangle whose area was measured, with a square drawn next to it. They were asked how many 1 cm squares could fit into the drawn rectangle, the dimensions of which were 5 cm and 6 cm. In the eliminatory part of the second task, the pupils were required to measure the length of a 10-centimeter-long line and a proper use of the ruler was recorded as well. *The last task* was designed to explore the strategies without the image showing square tiles and the rectangle, which implied a higher level of abstract thinking.

Results and discussion

1. Children's Covering Strategies

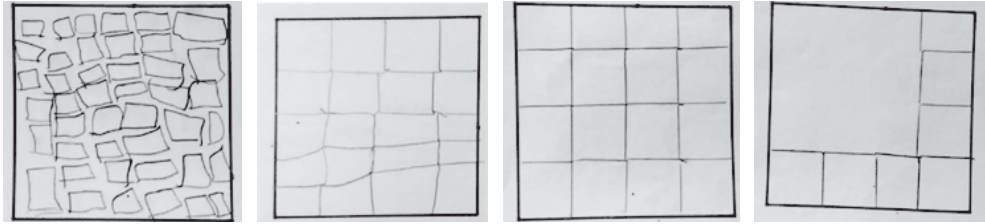
Task 1 Strategies. We identified four strategies used by pupils (Table 1). The physical action of covering with a square unit was the most commonly used strategy (Concrete covering, 13).

Table 1. *Frequencies of Children Using the Various Strategies Notice for Task 1 (N=44)*

Task 1	Total	Total	
		Correct count	Incorrect count
1.1. Incomplete covering	11		
1.2. Visual covering	9		
1.3. Concrete covering	13		
1.4. Measurement	11	25	19
Total	44		

Figure 1 shows typical pupil strategies.

Figure 1. *Examples of Strategies Notice for Task 1*



1.1. Incomplete covering

1.2. Visual covering

1.3. Concrete covering

1.4. Measurement

We observed that one pupil who used the Visual Covering strategy managed to calculate the correct total number of square tiles. The number of pupils who provided the correct answer was 25, whereas 19 pupils provided incorrect answers.

Task 2 Strategies. As the task was given visually, without a physical cardboard square, there was no covering strategy. Given that the efficacy of other strategies is the result of restructuring, when it comes to pupils' understanding of measuring the area of a figure intuitively, more correct answers were expected in Task 2 than in Task 1.

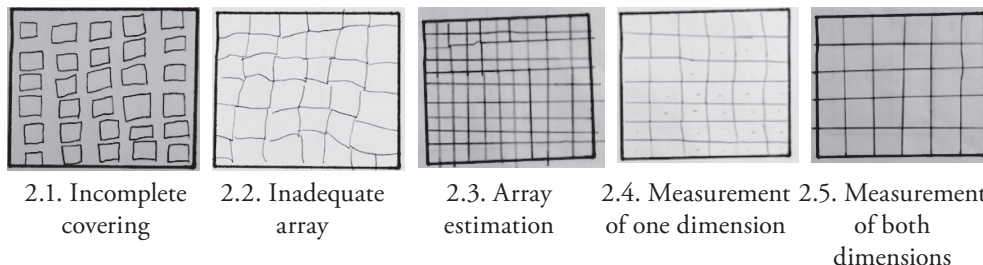
Out of a total of 25 pupils, 7 failed to solve the eliminatory part of the task involving the use of the ruler. Therefore, 18 pupils continued working on Task 2. Other students made mistakes when they started measuring from the first and not the zero point. Five strategies were identified (Table 2). Measurement of Both Dimensions was the most common strategy (10) and it was often used in Task 1 as well. All the pupils who used this strategy managed to solve the task correctly (10), while 8 pupils failed to do so.

Table 2. *Frequencies of Children Using the Various Strategies Notice for Task 2 (N=18)*

Task 2	Total	Total	
		Correct count	Incorrect count
2.1. Incomplete covering	1		
2.2. Inadequate array	0		
2.3. Array estimation	1		
2.4. Measurement of one dimension	6	10	8
2.5. Measurement of both dimensions	10		
Total	18		

Typical pupil strategies are presented in Figure 2.

Figure 2. *Examples of Strategies Notice for Task 2*



Task 3 Strategies. This task was in textual form only. Even the length of the side of the unit square was not of unit length (2 cm). The task was given to pupils who used the Measurement of Both Dimensions strategy in the previous, visual task (this was the only successful strategy). Array Estimation was the most frequently used strategy (4). All students who completed the task did it correctly (10).

Table 3. *Frequencies of Children Using the Various Strategies Notice for Task 3 (N=10)*

Task 3	Total	Total	
		Correct count	Incorrect count
3.1. Array estimation	4		
3.2. Measurement of one dimension	2		
3.3. Array drawn or implied	3	10	0
3.4. Array calculated	1		
Total	10		

2. Classification of Strategies into Developmental Levels

We drew upon the paper of Outhred and Mitchelmore (2000) and followed the idea of the existence of many developmental levels in pupils regarding the strategies of intuitive calculating of the area of a rectangle. By analyzing the obtained data and in line with the research objective, we realized that in our research sample, there were different strategies of different complexity and that they could be classified into different developmental levels. Some pupils use simpler strategies. These pupils left a lot of blank space between

the drawn square tiles or tended to overlay the square tiles extensively due to random and not systematic drawing. The other group of pupils was good at drawing units but made mistakes when drawing other shapes of different structures and numbers of units. Even the pupils who did the task correctly demonstrated strategies of different developmental levels. Based on all these observations, we identified five developmental levels, which corresponded to the findings of Outhred and Mitchelmore (2000). Table 4 illustrates pupils' achievement on each of the three tasks according to developmental levels, while Table 5 presents the data relative to pupils' age.

Table 4. *Estimated Frequencies Success Rates and Frequencies of Students Completing Each Task, by Developmental Level*

Task	Level 0: Incomplete covering	Level 1: Primitive covering	Level 2: Array covering, constructed from unit	Level 3: Array covering, constructed by measurement	Level 4: Array implied solution by calculation
Estimated frequencies success rates *					
1	11	9	13	11	0
2	1	2	7	8	0
3	0	0	1	8	1
Frequencies of students completing each task **					
1	0	1	13	11	0
2	0	0	2	8	0
3	0	0	1	8	1

**Note.* The data show the estimate of success in the entire sample (N=44), assuming that the pupils who did not offer a solution or did not do Task 2 or Task 3 are at level 0 or level 1 and they would not be successful on these tasks. **26 pupils did not solve Task 2 and 34 did not solve Task 3.

Table 4 shows the correlation between pupils' estimated success and the developmental level of the strategy. The superiority of Level 2 and Level 3 strategies was confirmed. Higher-level strategies were superior in the research of Outhred and Mitchelmore (2000). In our research, it was evident that Tasks 2 and 3 were solved using successful strategies, showing that pupils who used Level 2 and Level 3 strategies in Tasks 2 and 3 could generalize the strategies in order to understand the unusual measurement unit in Task 3.

Table 5. *Estimated Frequencies Success Rates and Frequencies of Students Completing Each Task, by Grade Level*

Task	Preschool	Grade 1	Grade 2	Grade 3
Estimated frequencies success rates*				
1	13	12	8	11
2	2	5	7	11
3	0	0	3	7
Frequencies of students completing each task **				
1	2	5	7	11
2	0	0	3	7
3	0	0	3	7

* *Note.* The data show the estimate of success in the entire sample (N=44), assuming that the pupils who did not offer a solution or did not do Task 2 or Task 3 would not be successful on these tasks. **26 pupils did not solve Task 2 and 34 did not solve Task 3.

Table 5 illustrates the relationship between the assessment of pupils' success rate and their age. The success rate in solving tasks increased with age, with pupils using more efficient strategies. For example, only second graders and third graders used Level 3 and Level 4 strategies. These pupils were not previously taught the area formula. Some of them were taught the array model of multiplication and encouraged by the previous tasks involving array counting and drawing. Still, they were able to use multiplication to calculate the total number of units.

3. The Conceptualization of Area

The five-level classification of strategies in Tasks 1-3 indicates a gradual development of pupils' ability to calculate the area of a rectangle either by using a picture or mental representation. There are principles P1-5, which are key to development and adopted in succession (Outhred & Mitchelmore, 2000):

- P1: The rectangle must be completely covered by the units, without overlaps or gaps.
- P2: The units must be aligned in an array with the same number of units in each row.

- P3: Both the number of units in each row and the number of rows can be determined from the lengths of the sides of the rectangle.
- P4: The number of units in a rectangular array can be calculated from the number of units in each row and in each column.
- P5: The length of a line specifies the number of unit lengths that fit along the line.

Implications for the Learning of Area

The results of our research confirm the results obtained by Outhred and Mitchelmore (2000) and indicate a relational understanding of the area formula that develops according to a model. The concepts of area measurement and covering are learnt in the last phases of the proposed model (Outhred & Mitchelmore, 2000): 1) early introduction of the concept of area measurement by area covering whose area is measured by a selected measurement unit and the research of the rectangular array covering or 2) researching the rectangular array covering as a problem *per se* by introducing the concept of area and using the acquired knowledge of area covering to measure the area of the rectangle. A parallel introduction of the concept of area and area covering is also possible.

We should note that 3 second graders and 7 third graders solved Task 2, while as many as 7 third graders pupils solved the most difficult Task 3. The results for Serbia are similar to the results obtained by Outhred and Mitchelmore (2000). Though the sample on which the research was repeated was small, second- and third grade pupils had a good intuitive foundation for area measurement. Therefore, the results could be generalized in favor of teaching area measurement to a larger number of pupils in initial mathematics instruction, that is, at an even younger age. As far as the model proposed by Outhred and Mitchelmore (2000) is concerned, we emphasize the following: 1) it is important that pupils understand spatial structure; 2) relational understanding of length measurement is essential; 3) pupils should link area measurement to length measurement and the concept of multiplication before they are taught the area formula and before they fully understand it.

Conclusions

There are many developmental levels in calculating the area of a rectangle. At a specific developmental stage, pupils are able to understand rectangular array covering as a means of measuring the area of a rectangle (Eames et al., 2020; Milošević, 2020; Milošević & Đokić, 2022). Developmental strategies are based on specific principles. The conceptualization of covering the area of a rectangular array plays a key role in pupils' understanding of rectangular area measurement.

Keywords: the meaning of area, rectangle's area, intuitive understanding, pupil strategies, measurement of a rectangle's area, Serbia.

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APPENDICES

Appendix 1. Tasks 1 and Task 2 from the linked interview (Outhred & Mitchelmore, 2000: 148)

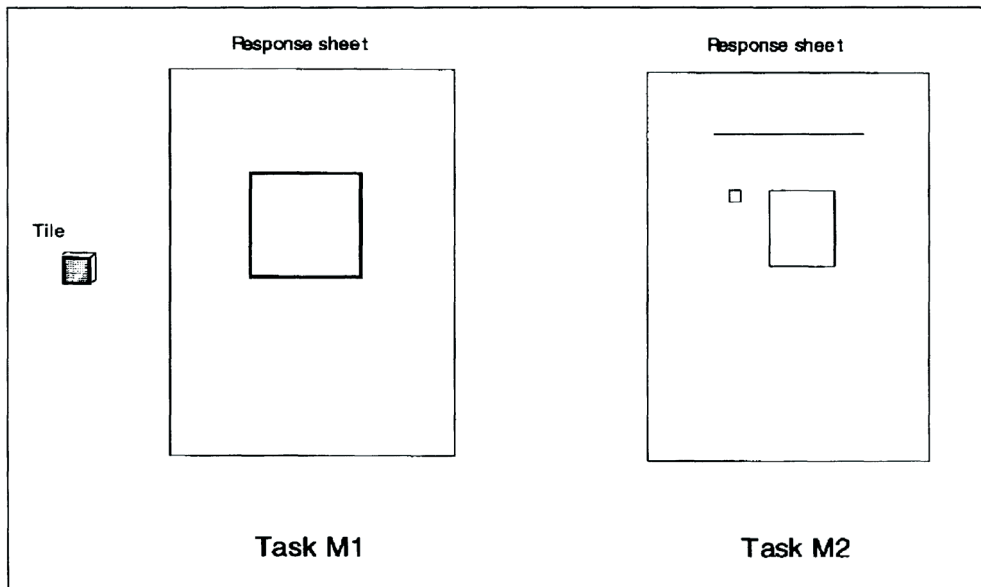


Figure 1. *The measurement tasks M1 and M2*